ADDIS ABABA SCIENCE AND TECHNOLOGY UNIVERSITY
SCHOOL OF GRADUATE STUDIES

COLLEGE OF ARCHETECT and CIVIL ENGINEERING

PLASTIC ANALYSIS OF UNBRACED PORTAL FRAMES OF STEEL STRUCTURES IN ADDIS ABABA

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APPROVAL PAGE

This Msc thesis entitled with **“*plastic analysis of unbraced portal frames of steel structures in addis ababa*”** has been approved by the following examiners in partial fulfillment of the requirement for the degree of Master of Science in **Structural Engineering**.

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CERTIFICATION

I, the undersigned, certify that I read and hear by recommend for acceptance by Addis Ababa Science and Technology University a dissertation entitled “plastic ***analysis of unbraced portal frames of steel structures in addis ababa”*** in partial fulfillment of the requirement for the degree of Master of Engineering in Structural Engineering.

Dr. Suresh Borra

Principal Advisor

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Sendek Amare Aragie, declare that this thesis is my own original work that has not been presented and will not be presented by me to any other University for similar or any other degree award.

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DEDICATION

In loving memory of my Mother

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List of symbols:

$K_T -$ Terrain factor

$C_r(z)$ $-$Roughness coefficent

$C_t(z)$ $-$ Topography coefficient

C_{DIR} $-$ The directional factor

C_{TEM} $-$ The temporary (or Seasonal factor)

$\rho -$ Air Density

$V_{ref,o}$ $-$ Is the basic value of the reference wind velocity

M_{cr} $-$ Elastic critical moment corresponding to lateral torsional buckling.

M_{br} $-$ required flexural strength of torsional bracing.

$\psi -$ ratio of the moment at the ends of the laterally unsupported length of a beam.

α_{LT} $-$An imperfection factor

$\chi -$ stress reduction factor due to buckling under consideration

χ_{LT} $-$strength reduction factor

$\phi -$ Strength or resistance reduction factor; cumulative distribution function ; solidity ratio; inclination of the tension field stress in the web; configuration factor ; angle of twist.

β_{MLT} $-$ Equivalent uniform moment factor for lateral torsional buckling.

$L_m -$ Maximum distance from the restraint at plastic hinge to an adjacent restraint (limiting distance)

l_s $-$Length between points of lateral support

K_w $-$warping restraint factor

K_y, K_z $-$ moment amplification factor about respective axes

C_1 $-$ Equivalent uniform moment factor

C_{pe} −External pressure coefficient

C_{pi} −Internal pressure coefficient

I_t − St. Venants torsional constant

I_w − Warping constant

S_R −Reduced plastic modulus

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Abstract:

The versatile nature of the steel having ductility facilitating redistribution of moments at the ultimate limit state, therefore this character utilizing the underutilized areas in the elements of the frame, making the origination of plastic analysis. The areas which reach the maximum capacity forms the plastic hinges and ultimately leads to the formation of mechanism. In the frame analysis either elastic analysis or plastic analysis are used. But the elastic analysis does not give the ultimate capacity. Under the plastic analysis concept the total area under the elastic and plastic plateau areas of the stress –strain curve is utilized without changing the dimensions of the sections. This is facilitating the effective utilization of the member and consequently economy in the design and construction would occur. Particularly in developing countries where the resources are scarce and the FOREX is under heavy pressure designing the industrial buildings with plastic method could reduce the negative effects. The plastic analysis has proved and practiced as a reliable method for designing not only for industrial buildings also for commercial buildings (Long since it has been practiced in Unite Kingdom)in the developed world. In the current scenario of Ethiopia the plastic analysis is an alternative to elastic analysis which could provide economy to the structures design and construction.

In general, constructions constructed using Steel structures in Ethiopian has not yet developed. This can be due to many factors. But to name a few, unavailability of the profile sheet locally and limited exposure to steel profile design are some factors. But recently, especially in the capital, Addis Ababa, it is not uncommon to see some industrial garages are made from steel portal frames.

In relation to this an industrial building located in Addis Ababa which is constructed with Gable frame has taken for the analysis by using plastic analysis. The gable frame is having the width 40m and height is 8m analyzed by considering the wind load showed that the members of the gable frame i.e, column and rafters are carrying more load in plastic analysis rather than elastic analysis even though the sizes of the members are not changed in these types of analysis

.Key words: Plastic , Elastic, Plastic hinge , moment redistribution, Gable frame.

Chapter 1

Introduction

1.1 Background:

The single story clear span building is constant demand for ware houses, factories and many other purposes. The clear internal appearances makes it much more appealing than a trussed roof building and it also requires less maintenance. The portal may be of 3-pinned, pinned base or fixed base construction. The pinned base portal is the most common type adopted because of the greater economy in foundation design over fixed base type.

Steel is by far most useful material for building construction in the world. Today steel industry is the basic or key industry in any country. It strength of approximately ten times that of concrete, steel is the ideal material of modern construction. It's mainly advantages are strength, speed of erection, prefabrication, and demountability. Structural steel is used in load-bearing frames in buildings, and as members in trusses, bridges, and space frames. Steel, however, requires fire and corrosion protection. In steel buildings, claddings and dividing walls are made up of masonry or other materials, and often a concrete foundation is provided. Steel is also used in conjunction frame and shear wall construction. Due to its large strength to weight ratio, steel structures tend to be more economical than concrete structures for tall buildings and large span buildings and bridges. Steel structures can be constructed very fast and this enables the structure to be used early thereby leading to overall economy steel offers much better compressive and tensile strength than concrete and enables lighter constructions.

To get the most benefit out of steel, steel structures should be designed and protected to resist corrosion and fire. They should be designed and detailed for easy fabrication and erection. Good quality control is essential to ensure proper fitting of the various structural elements. The effects of temperature should be considered in design. Steel structures are ductile and robust and can withstand severe loadings such as earthquakes. Steel structures can be easily repaired and retrofitted to carry higher loads. Steel is one of the friendliest environmental building materials – steel is 100% recyclable.

To prevent development of cracks under fatigue and earthquake loads the connections and in particular the welds should be designed and detailed properly. Special steels and protective measures for corrosion and fire are available and the designer should be familiar with the options available. Since steel is produced in the factory under better quality control, steel structures have higher reliability and safety.

HISTORICAL DEVELOPMENT OF STEEL

Steel has been known from 3000 BC steel was used during 500-400 BC in china and then in Europe. In India the Ashoakan pillar made with steel and the iron joints used in Puri temples are more than 1500 years old.

The large-scale use of iron for structural purposes started in Europe in the latter part of the eighteen century.The first major application of cast iron was in the 30.4 –m-span Coalbroakadale Arch Bridge by Darby in England, constructed in 1779 over the river Severn. The use of cast iron was continued up to about 1840. In 1740, Abraham Darby found a way of converting coal into coke, which revolutionized the iron –making process.

In 1784 Henory Cort found way of wrought iron, which is stronger, flexible, and had a higher tensile strength than cast iron. During 1829 wrought iron chains were used in Menai Straits suspension bridge designed by Thomas Telford and Robert Stephenson‟s Britannia Bridge was the first box girder wrought iron bridge. Steel was first introduced in 1740,but was not available in large quantities until Sir Henry Bessemer of England invented and patented the process of making steel in 1855. In 1865,Siemens and Martin invented the open –hearth process and this was used extensively for the production of structural steel. Companies such as Dorman Long started rolling steel I-section by 1880.Riveting was used as a fastening method until around 1950when it was superseded by welding. Bessemer’s steel production in Britain ended in 1974 and last open –hearth furnace closed in 1980.The basic oxygen steel making (BOS) process using the CD converter was invented in Austria in 1953.Today we have several varieties of steel.

TYPES OF STRUCTURAL STEEL

The structural designer is now in a position to select structural steel for a particular application from the following general categories.

a) Carbon steel (IS 2062)

Carbon and manganese are the main strengthening elements. The specified minimum ultimate tensile strength for these varies about 380 to 450 MPa and their specified minimum yield strength from about 230 to 300MPa(IS 800:2007)

b) High –strength carbon steel

This steel specified for structures such as transmission lines and microwaves towers. The specified ultimate tensile strength, ranging from about 480-550 MPa, and a minimum yield strength of about 350-400 MPa.

c) Medium-and-high strength micro alloyed steel(IS 85000)

This steel has low carbon content but achieves high strength due to the addition of alloys such as niobium, vanadium, titanium, or boron. The specified ultimate tensile strength, ranging from about 440-590 MPa,and a minimum yield strength of about 300-450 MPa.

d) High –strength quenched and temperature steels(IS 2003):This steel is heat treated to develop high strength. The specified ultimate tensile strength, ranging from about 700-950 Mpa, and a minimum yield strength of about 550-700 Mpa

e) Weathering steels

This steel low-alloy atmospheric corrosion –resistant .They have an ultimate tensile strength of about 480 MPa and a yielded strength of about 350 MPa.

f) Stainless steels

This steel is essential low-carbon steel to which a minimum of 10.5% (max 20%) chromium and 0.5% nickel is added.

g) Fire-resistant steels

Also called thermo-mechanically treated steels, they perform better than ordinary steel under fire.

From 20 to 25 years a considerable research has done on ultimate strength of the steel structures. These studies have revealed possibilities for the maximum strength (plastic strength) of the steel can be considered as a basis for design. But it is only in recent years tests on large size of structural members and frames have conducted and adequate analytical techniques developed to make the method of practical use. The practical usage of this method had been taken place in the developed world long back. It's application is having more relevance in developing countries too, where they are having scarce resources this method is not initiated yet. Therefore the country like Ethiopia where fast industrialization is taking place in the environment of lacking basic industry in steel manufacturing and the scarcity of the Forex prevailed, application of the plastic method has more relevance.

Many investigators have contributed for the development of plastic analysis. But recent developments are due to the efforts of W.Prager, P.S.Symonds and D.C. Drucker at the Brown University and J.F.Baker, J.W.Roderick, M.R. Horne and B.G Neal at Cambridge University.

By plastic analysis the engineer is able to find the true load carrying capacity of the structure. And plastic design has an appeal on the basis of its simplicity. Also it eliminates the most time consuming elastic analysis. Further imperfections that seriously affect elastic limit strength of a structure (such as sinking of supports, differences in flexibility of connections, spreading of supports and residual stresses) have little or no effect upon the maximum plastic strength.

Finally these techniques promise to produce a structure having substantial savings through the more economic and efficient use of steel and savings in design office time.

Therefore it can be expected that plastic design can find considerable application particularly in continuous beams, industrial frames and also in tier buildings. As a matter of fact it has been reported that more than 250 industrial frames have been designed in England by using plastic method. Simultaneously school building and five story buildings also designed are some of the examples designed under this method.

1.2 Statement of the problem:

With recent technological developments plastic design methods which were limited to one and two story rigid frames have been extended to unbraced multi-story frames. Systematic procedures for the application of plastic design in proportioning the members of such frames have been developed and are available in the current literature.

Plastic analysis gives the economical sections and in Ethiopia the portal frames are designed by the elastic analysis which is uneconomical where the resources are scarce.

Thus, this study is intended in order to prepare plastic analysis of portal frames that will be used as a reference for designers and local manufacturers in the country.

Therefore, there has been an increasing demand for the development of a practical method for the analysis of portal steel frames, on which a sound design method would be based.

1.3 Objective:

1.3.1 General objective:

To standardize the portal frames analysis and design by plastic analysis. The objective of this dissertation is to develop an exact analytical method for predicting the complete elastic-plastic loading and unloading behavior of an unbraced portal steel frames, found in adds Ababa zone areas , subjected to non-proportional combined loading where gravity loads are constant and lateral loads vary. Based on the analytical approach developed in this dissertation, a procedure will be developed for determining the approximate load-deflection behavior on regions of portal steel frame.

1.3.2 Specific objective:

Comparing the plastic analysis and elastic analysis members for portal frames

Design according to the EBCS-3

Development moment- curvature graphs

1.4 Methodology:

Analysis of the pitched roof has done from the wind load point of view in order to find the loads on the portal frame/ gabled frame

Rigorous literature review in order to understand the elastic analysis and plastic analysis and the methods of design of the portal frame/ gable frame by using elastic analysis and plastic analysis

For application of design of the portal frame through understanding of the EBCS-3 draft is required

1.5 Limitation of the project:

$M - \emptyset$ Curves development requires the software. Manually developing is not possible. This is the objective specified is not achieved.

1.6 Literature review:

1) According to the "plastic analysis of Steel structure" paper published by the M. Rogac, M. Knezevic , M. Cvetkovska Civil Engineering Faculty, Podgorica, Montenegro, Plastic analysis includes large deformations, so there is a question of justification of the basic assumptions of plastic analysis, especially of Bernoulli's hypothesis of plane cross sections
2) WHY PLASTIC DESIGN by Lynn S. Beedle Prepared for delivery to AISC-USC Conference on "PLASTIC DESIGN IN STRUCTURAL STEEL " Los Angeles, California. Plastic design has come of age. Considerable literature is available in the form of lecture notes, reference books and various technical proceedings .Mr. Higgins and Mr. Estes of the American Institute of Steel. Construction is nearing completion of a manual on plastic. Design which will afford the designer with even more specific examples and techniques. Thus engineers in this country U.S.A will.be able to join with those in England and in Canada who have already applied plastic analysis to their design problems.
3) Elastic-plastic analysis and design of un braced multi-story steel frames, Ph.D. Dissertation, May 1966 B. P. Parikh- The method of second-order elastic-plastic analysis is presented in this discussion and is found to be the best method for predicting the true behavior of multi-story frames acted upon by gravity loads and lateral loads.

4) New Plastic Approach to Plastic Analysis of Steel Structures by Eshan Dehghani, Sajad A. Hamidi, Faribroz M. Tehrani, Aastha Goyal, Rasoul Mirghaderi—They focused on the propagation effects of the plasticity in both section and length of the element.
5) Historical Development:

The application of plastic analysis to structural design appears to have been initiated by Dr.Gabor Kazinczy, a Hungarian, who published results of his test of clamped girders as early as 1914.He also suggested analytical procedures similar to those now current and designs of apartment type buildings were actually carried out

In his strength of materials , Timoshenko refers to early suggestions to utilize ultimate load capacity in the plastic range and states “ Such procedure appears logical in the case of steel structures submitted to the action of stationary loads, since in such cases a failure owing to the fatigue of metal is excluded and only failure due to the yielding of metals has to be considered.

Early tests in Germany were made by Maier-Leibnitz who showed that the ultimate capacity was not affected by settlement of supports of continuous beams. In doing so he corroborated the procedures previously developed by others for the calculation of maximum load capacity.

The efforts of van den Borek in Germany and J.F Baker and his associate in Great Britain to actually utilize the plastic reserve strength as a design criterion are well known.

Progress in theory of plastic structural analysis(particularly that at Brown University)has been summarized by Symonds and Neal. A survey of design trends by winter, discusses briefly many of the factors Germaine to plastic design.

For more than ten years the American Institute of steel construction , the welding Research Council, the Navy Department and the American Iron and steel Institute have sponsored studies at Leigh University. These studies have featured not only verification of this method of analysis through appropriate tests on large structures , but have given particular attention to the conditions that must be met to satisfy important secondary design requirements.

Summary:

From the literature review of the journals and the researchers the analysis and design of the sections under plastic is having more load carrying capacity. But in order to attain that capacity the frames requires more restraints in order to form the plastic hinges at the specified locations and to have the moment redistribution in the sections.

Chapter 2 Plastic Method

2.1 Comparison of Elastic and plastic analysis:

Elastic analysis is the most common method of analysis for general structures, but will usually give less economical portal structures than plastic analysis. Plastic analysis is not used extensively in continental Europe, even though it is a well proven method of analysis. However plastic analysis is used for more than 90% of portal structures in the U.K and has been in use for 40 years. The elastic design method also termed as allowable stress method (or working stress method) is a conventional method of design based on the elastic properties of steel. This method of design limits the structural usefulness of the material up to a certain allowable stress which is well below the elastic limit. The stresses due to working loads do not exceed the specified allowable stresses, which are obtained by applying an adequate factor of safety to the yield stress of steel. The elastic design does not take into account the strength of the material beyond the elastic stress. Therefore the structure designed according to this method will be heavier than that designed by plastic methods but in many cases elastic design will also require less stability bracing.

In the method of plastic design of a structure the ultimate load rather than the yield stress is regarded as the design criterion. The term plastic has occurred due to the fact that the ultimate load is found from the strength of the steel in the plastic range. This method is also known as method of load factor design. The strength of the steel beyond the steel beyond the yield stress is fully utilized in this method. This method is rapid and provides a rational approach for the analysis of the structure.

Traditionally manual calculation methods were used for a plastic analysis (The so called graphical method or the virtual work method etc.) The elastic perfectly plastic model assumes that the members deform as linear elastic elements until the applied moment reaches the full plastic moment$M_{p.}$ The subsequent behavior is assumed to be perfectly plastic without strain hardening.

The economy of plastic analysis also depends on the bracing system because plastic redistribution imposes additional requirements on the restraint to members. The overall economy of the frame might therefore depend on the ease with which the frame can be restrained.

Plastic design requires that the last plastic hinge occurs at or above the design load level. If both elastic and plastic designs satisfy the same design loading the plastic design method requires a lighter structures with smaller size by utilizing the reserve strength of the structure. It is noted that for a structure with a high degree of static indeterminacy the reserve strength is large. Therefore the benefit of using the plastic design is greater for structures with high degree of static indeterminacy .However for determinate structures that require only one plastic hinge to induce a collapse, there is no difference between elastic and plastic analysis.

Conditions for correct Plastic analysis	Conditions for correct elastic analysis.
1) Mechanism: a) The limit load is reached when the correct mechanism forms b) The number of plastic hinges developed should be just sufficient to form a mechanism. c) Additional deformations are possible without load increase.	1)**Continuity: (compatibility)** The deformations are proportional to the loads
2) Equilibrium: The sum of all forces and moments are equal to zero $\sum F_X = 0, \sum F_Y = 0 , \sum M_{XY} = 0$	**2)Equilibrium:** The sum of all forces and moments is equal to zero
3) Plastic Moment: The moment nowhere exceeds plastic moment $\lvert M \rvert \leq M_{plastic}$	3)**Elasticity : Yield moment** The moment nowhere exceeds yield moment $\lvert M \rvert \leq M_y$

Table 2.1 Comparison of Elastic and plastic

The basic conditions that are to be satisfied for any structure in elastic and plastic analysis are shown in the above table. If all the three conditions are satisfied , the lowest plastic limit load (A unique value) is obtained. If only the equilibrium and mechanism conditions are satisfied (this forms the basis for the mechanism method of plastic analysis), an upper bound solution for the true ultimate load is obtained. A lower bound solution for the true ultimate load is obtained when equilibrium and plasticity conditions only are satisfied (statical method of plastic analysis)

Methods of plastic analysis:

Using the principle of virtual work and the upper bound and lower bound thermos , a structure can be analyzed for its ultimate load by any of the following methods:

i) Static method and
ii) Kinematic method.

Principle of virtual work says that this is simply a method to express the equilibrium condition. While applying this method to determine the moment at collapse , an arbitrary displacement is assumed at a plastic hinge location (The arbitrary displacement must be one for which only the internal moments at the plastic hinges contribute to the internal work) and the work done by the external and internal forces is equated. This is accomplished by allowing rotations of the structure only at points of simple support and at points where plastic moments are expected to occur in producing the mechanism.

The plastic analysis of structures is governed by three thermos , which are as follows. The static or lower bound theorem states that a load computed on the basis of an assumed equilibrium moment diagram, in which the moments are nowhere greater than the plastic moment , is less than , or at the best equal to, the correct collapse load. Hence the static method represents the lower limit to the true ultimate load and has a maximum factor of safety. The static thermo was first suggested by kist and its proof was given by Gvozder, Greenberg, and Horne(Horne 1979). The kinematic or upper bound theorem states that a load computed on the basis of an assumed mechanism will always be greater than , or at the best equal to, the correct collapse load. Hence the kinematic method represents an upper limit to the true ultimate load and has a smaller factor of safety compared to the static method. A proof of this theorem was provided by Gvozder, Grrenberg and Prager(Horne 1979). The upper and lower bound theorems can be combined to produce the uniqueness theorems at the same time is the correct collapse load.

Therefore in the mechanism or kinematic method , a mechanism is assumed and virtual work equations are assumed to determine the collapse load. The number of independent mechanisms (n) is related to the number of possible plastic hinge locations (h) and the number of degrees of redundancy ® of the frame by the equation $n = h - r$

In the statical or equilibrium method , an equilibrium moment diagram is obtained such that the moment at any section is less than or equal to the plastic moment capacity. Even though the equilibrium method gives a lower bound solution the virtual method is often used due to its simplicity of application in comparison with the equilibrium method.

If the upper and lower bound solutions obtained by the mechanism and statical methods coincide or are sufficiently close, then the assumed plastic hinge locations are correct. If, however , these bounds are not precise enough , then the location of the assumed hinge should be modified(an indication of this will be provided by the bending moments determined in the static analysis) and the analysis is repeated.

Basis of Plastic Theory:

Tests on actual buildings was carried out in the mid-1930s in USA and UK made it clear that a steel frame behaves differently from the assumptions made in conventional simple elastic theory. These tests showed that the real factor of safety of structural elements was very different from what has been assumed. This is because in the elastic design method, the member capacity is based on the attainment of yield stress. However steel has unique property called ductility, because of which it is able to absorb large deformations beyond the elastic limit without fracture. Due to this property, steel possesses reserve strength beyond its yield strength. The method which utilizes this reserve strength is called the plastic method of analysis.

The plastic theory makes the design process more rational , since the level of safety is related to the collapse load of the structure and not the apparent failure at one point. The concept of design based on ultimate load was first developed in Hungary in 1914 by Dr.Gabor Kazinczy. He carried out tests on fixed ended beams and came to the conclusion that failure took place only when yielding occurred at the three cross sections at which the hinging action takes place. The German engineer Maier-Leibnitz showed that the ultimate capacity of continuous beams is not effected by the settlement of supports. Lord J.F baker of Cambridge university along with proof. Horne and Prof. Hayman evolved the method of simple plastic theory , which resulted in collapse loads being considered as the design criteria(Horne 1979). In USA , Van den Broek wrote the first published paper on plastic theory in 1939. Subsequent work at Lehigh University resulted in several publications (eg., Beedle 1958: ASCE 1971). It has been found that plastic methods are easier to apply than elastic methods in some cases. However , the frames and their components must be checked rigorously for overall and local stability because of the high strains that might have to be endured

Although the entire section yields in tension and the compression members yield under the ultimate load , plastification of the material across the depth as well as along the length of the member takes place in the beam under the ultimate transverse load, and the members can behave elasto-plastically. A plastic hinge concept was also evolved to simplify computations (Davison & Owens 2004).

2.2 Loads and construction:

2.2.1 In plane the portal resists the following loads by rigid frame action

i) Dead and imposed loads acting vertically

ii) Wind causing horizontal loads on the walls and generally uplift loads on the roof slopes.

2.2.2 Construction: Main features in modern portal construction.

Columns: Uniform universal beam section

Rafters: Universal beams with Haunched ends ,usually of sections 30to 40% lighter than the columns.

Eaves and Ridge joints: Site bolted joints using Grade 8.8 bolts , where the Haunched ends of the rafters provide the necessary lever arm for design .Local joint stiffening is required.

Base: Normally pinned with two or four holding down bolts.

Purlins and sheeting rails: Cold rolled sections spaced at not greater than 1.75m to 2m centers.

Stays from purlins and rails: These provide lateral support to the inside flange of portal members.

Gable frame: A braced (not a rigid) Frame at the gable ends of the buildings

Bracing: Provided in the end bay in roof and walls.

Eaves and ridge ties: May be provided in larger span portals, though now replaced by stays from purlins or sheeting rails.

Design Outline: The code (EBCS-3, 2013) states that either elastic or plastic design may be used. Plastic design gives the more economical solution and is almost universally adopted. The design process of portal consists of

Analysis: The methods of frame analysis at ultimate limit state falls broadly in two categories Elastic analysis and plastic analysis. The later term covers both rigid plastic and elasto-plastic

Design of members: Taking into account of flexural and lateral torsional buckling with provision of restraints to limit out of plane of buckling. And sway stability check in the plane of the portal.

Joint design: With provision of stiffeners to ensure all parts are capable of transmitting design actions.

Serviceability check: For deflections at eaves.

2.3 Portal Analysis:

The most convenient manual method of analysis is to use formulae from the steel designers manual (Procedures for elastic and plastic design are set out in BS 5950). In this a general load case can be broken down into separate cases for which solutions are given and then these results are recombined. Computer analysis is the most convenient method to use particularly for wind loads and load combinations. The output gives design actions and deflections. Bespoke software for portal frame design is widely available, which will undertake elastic-plastic analysis, allow for second order effects, verify members and also verify connections.

The Bending moment diagram for the dead and imposed load case is given below. This shows the inside flange of the column and rafter near the eaves to be in compression and hence the need for lateral restraints in those areas.

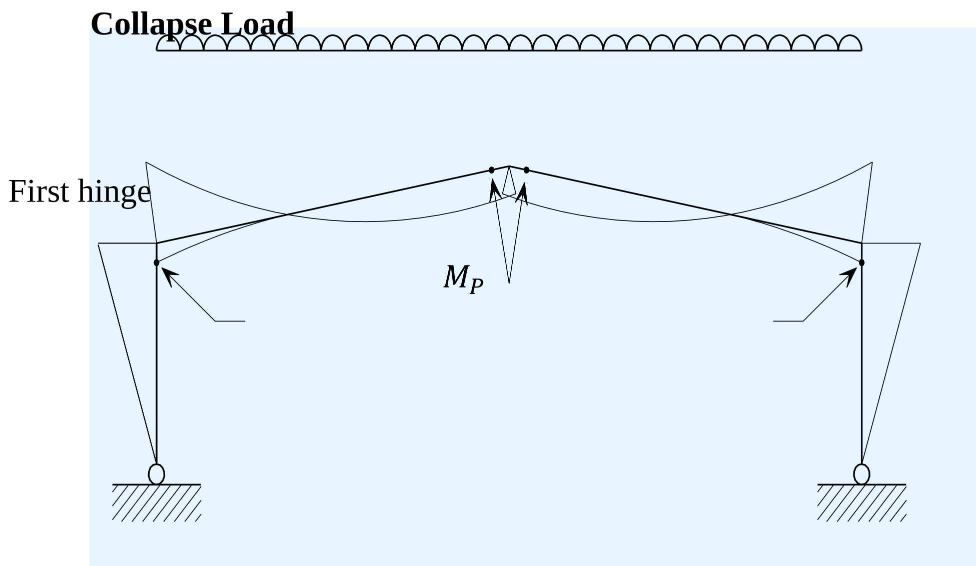


Fig 2.1-Plastic Moments and Hinges

An idealized “plastic” bending moment diagram under symmetrical vertical loads is shown in above figure for a symmetrical portal. This shows the position of the formed plastic hinges for the plastic collapse mechanism. The first hinge is to form normally adjacent to the haunch (shown in the column in this case). Later, depending on the proportions of the portal frame, hinges form just below the apex, at the point of maximum sagging moment

From an elastic analysis of a frame with pinned bases a typical bending moment diagram is shown in below figure. In this case, the maximum moment (at the eaves) is higher than that calculated from a plastic analysis. Therefore the column and the haunch, both have to be designed for these larger bending moments. The haunch may be lengthened to around 15% of the span, to accommodate the higher bending moment

Working Load

$M_H > M_S$

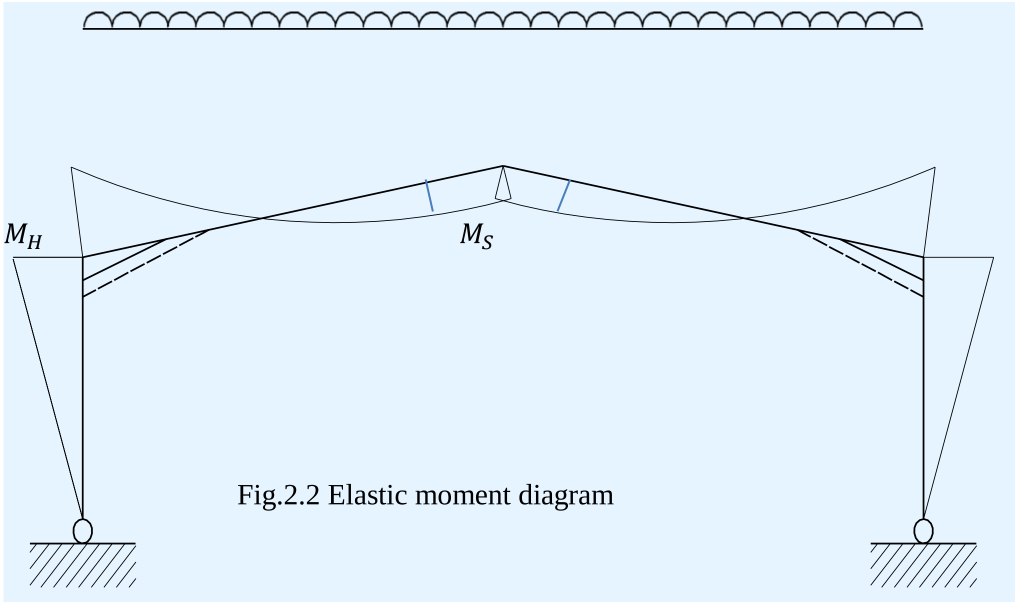


Fig.2.2 Elastic moment diagram

2.4 General requirements for utilizing plastic design:

Generally codes allow the use of plastic analysis only where the loading is predominantly static and fatigue is not a design criterion.

Codes put limitations which are intended to ensure that there is a sufficiently long plastic plateau to enable a hinge to form and that the steel will not experience a premature strain hardening. In clause 5.3.3, BS 5950 prescribes the following restrictions on the properties of the stress-strain curve for steels used in plastically designed structures.

i) The yield plateau(horizontal portion of the curve) is greater than six times the yield strain.
ii) The ultimate tensile strength must be more than 1.2 times the yield strength.
iii) The elongation on a standard gauge length is not less than 15%

On Ductility: Australian (AS4100) and United States (AISC) design codes specify that the yield strength of the steel for plastic design cannot exceed 450 Mpa as ductility becomes a concern when steels of higher yield strength are used.

In ultimate limit state requirements—Most rules for plastic design were developed many years ago when rigid plastic theory (Clause 5.4.3(5), EBCS-2013-Rigid plastic analysis) was commonly used for analyzing and designing simple to moderately complex structures. The elasto plastic analysis method introduced enables the design of structures with virtually any degree of complexity. Therefore care must be taken when applying these rules to complex structures. for example for some complex structures it may be necessary to check rotational capacity in plastic hinges even after all design code requirements are satisfied because of the complicated interaction between yielding of steel material and local buckling , most design rules , many of them empirical apply specifically to standard structural sections with double or mono symmetry such as I-sections, box sections, channels and circular hollow sections.

2.5 Importance of $M - \emptyset$ relationship:-

Curvature($\emptyset$) at a given stage is obtained from particular stress distribution .Corresponding moment value is obtained by integration of stress areas. Even though curvature is a measure of strain distribution the stress distribution diagram is used since, in the elastic range the stress varies linearly with strain

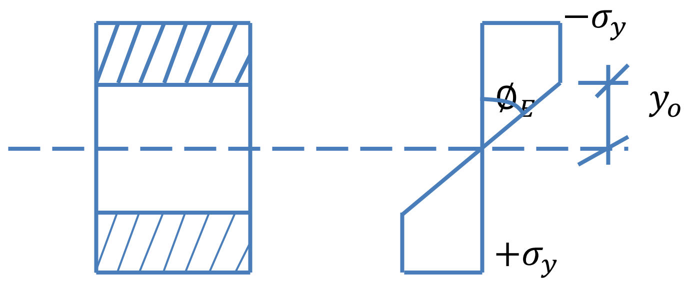


Fig.2.3. Stress distribution diagram

$\emptyset = \frac{\sigma_y}{E y_0}$ -- Eq-1

$\emptyset_y = \frac{\sigma_y}{E\frac{d}{2}}$ –Eq-2

$M - \emptyset$ Curve is basic to plastic analysis. It is the characteristic of plastic hinge. In addition to providing a measure of strength it has two fold role.

i) Characterizes rotation capacity of structural element ---i.e, ability of a structural member to rotate at near maximum moment.

ii) It is foundation of deformation computations, $\emptyset$ –diagram replaces $\frac{M}{EI}$ diagram in deformation analysis.

Plastic design is concerned directly with load carrying capacity. Idealization of $M - \emptyset$ curve makes possible application of slope deflection equations to estimate deflections beyond elastic limit and at ultimate load. Method applied to computation of required rotation capacity at hinges.

However problem of deflections is not critical to plastic design since a structure proportioned by plastic methods has restraining moments that are not present in conventional "Simple beam "design. The frequent result is that "simple beam" deflection is usually greater than that of a structure designed by plastic methods.

Slope deflection equation:

The following form of slope deflection will be used (clockwise M and θ are +ve)

$\theta_A = \theta'_A + \frac{\Delta}{L} + \frac{L}{3EI}(M_{AB} - \frac{M_{BA}}{2})$ –Eq-3

θ'_A –Simple beam end rotation

Δ – Vertical deflection with continuity assumed at section "A"

While the element subjected to rotation at maximum moment, the element should ensure the stability. Along with the elements stability frame stability also important for safety of the frames and eventually for the structure.

2.6 Stability:

The concept of stability as it applies to structural systems may be understood beat by considering the conditions of equilibrium. If a structural system that is in equilibrium is disturbed by a force, it has two basic alternatives when the disturbing force is removed.

a) It will return to its original position, in which case we refer to the system as being stable.
b) It will continue to deform as a consequence be incapable of supporting the load it supported before the disturbance occurred, in which case the system is called unstable.

Instability thus characterized as a change in geometry, which results in the loss of the ability to support load. Stability, specifically the loss of ability to support load, is an extremely important consideration in the development of the limit states design criterion. Thus buckling may be defined as a structural behavior in which a mode of deformation develops in a direction or plane perpendicular to that of the loading which produces it; such a deformation changes rapidly with increase in the magnitude of the applied loading . It occurs mainly in members or elements that are subjected to compressive forces.

While using plastic design, it is assumed that plastic deformation can take place without the geometry of the structure changing to such an extent that the conditions of equilibrium are significantly modified. Such changes in geometry can arise at three levels, namely (Horne & Morris 1981).

a) Deformation within the cross section of the member (resulting from local buckling in the plate elements constituting the web or flange)
b) Displacements within the length of the member relative to straight lines drawn between corresponding points of the end sections (due to the bending and/ or twisting of the member), and
c) Overall change of the geometry of the structure, causing the joints to displace relative to each other (e.g, the sway deformation in multi-story frames).

These three levels of deformation are thus associated, respectively, with local, member and frame instability. However, in plastic design it is assumed that plastic deformations, leading to some redistribution of stresses and bending moments, can take place before instability sets in and this means that the theoretical elastic instability load should be significantly above the plastic limit load.

There are number of special considerations while attempting a plastic design. For plastically designed frames three stability criteria have to be considered for ensuring the safety of the frame. These are

i) General frame stability
ii) Local Buckling criterion
iii) Restraints

i) General Frame Stability:

Usually under loading all structures move. But in some cases the movement of the structure is sufficient to drop the factor of safety (clause 5.1.3, 5.5.3.2 and 5.5.3.3 of BS 5950).Therefore the designer has to take into account the load carrying capacity in checking the structure.

ii) Local Buckling criterion:

At the location of a plastic hinge there is a considerable strain and at ultimate load this can reach several times the yield strain. Under these conditions sections will buckle or moment capacity will drop considerably, if in no circumstances should sections not complying with the plastic section classification limits in the code be used in locations where there are plastic hinges. Otherwise there is a real risk of premature reduction in the moment capacity of the member at the hinge location.

Flange stability:

One of the major assumptions in plastic theory and design is that the beam is supported continuously laterally to prevent the failure of the compression flange by lateral buckling. Therefore this condition should be translated in practice for the design to be valid. Note that plastic hinges require a certain amount of ductility in addition to their strength requirement—rotation capacity is a measure of this ductility(rotation capacity may be defined as that capacity which a given cross sectional shape can accept at the plastic moment without premature failure occurring). Thus a section should ideally exhibit a rotation capacity that corresponds to as train equal to the strain hardening strain $\varepsilon_{s,}$. Lateral instability can prevent a member from maintaining its nominal M_P value during hinge rotation, since yielding dramatically reduces the resistance of a member to lateral buckling; the moment sustained by the section reduces with increased rotation instead of remaining constant. To prevent this, members should be adequately braced against both lateral and torsional displacements at the positions at which hinges assumed in the failure mechanism.

The code (IS800) suggests that torsional restraints against lateral buckling should be provided at all plastic hinge locations. BS:5950-1:200 suggests that both flanges should have lateral restraints at each plastic hinge location to resist a force equal to 2.5 percent of the force in the compression flange. When it is not practicable to provide to provide such restraints at each plastic hinge location, it should be provided within a distance of $D/_2$, of the plastic hinge location , where "D" is the total depth of the section. However, for the hinge that forms last, this requirement is not required since it needs only just to attain the M_P. Value for the mechanism to be considered complete. Hence the code (IS800) states that the torsional restraint requirement need not be met at the last hinge to form, provided it can be clearly identified.

Within a length equal to the member depth , on either side of the plastic hinge location, the following restrictions should be applied to the tension flange.

a) Holes if provided, should be drilled, If punched they should be punched 2mm under sized and reamed.
b) All sheared or hand cut edges should be finished smooth by grinding, clipping or planning.

iii) Restraints:

In order to ensure that the plastic hinge position does not become source of premature failure during the rotation, therefore torsional restraint should be provided at the plastic hinge locations.

Stiffeners at plastic hinge locations: Web stiffeners should be provided at points where a concentrated load is applied within $D/_2$, of a plastic hinge location, which exceeds 10% of the shear capacity of the member (checked as per the provisions of the beams). The stiffener should be provided within a distance of half the depth of the member , on either side of the hinge location, and designed to carry the applied loads. If the stiffeners consist of plates , then the outstanding width to thickness ratio $b/_t$ should not

exceed the values given for plastic section . When other sections are used , the ratio $\left(\frac{I_{so}}{I_t}\right)^{0.5}$ should not exceeds the values given for the plastic section ,where I_{so} is the second moment of area of the stiffener about the face of the element perpendicular to the web and I_t is St. Venants torsion constant of the stiffener.

Members must be checked for the combined effects of axial load and buckling. In plane buckling is buckling about major axis of the member, there are no intermediate restrains when considering in-plane buckling of a member in a portal frame. Out-of-plane buckling concerns about the minor axis of the member .In a portal frame he secondary steel work can be used to provide restraints, and so increase the buckling resistance.

The element when reaches to its' maximum capacity under loading condition it should not be subjected to the buckling. Therefore ensuring the load carrying capacity of the elements requires buckling checks

2.6.1 Buckling resistance in Code:

The verification of buckling resistance of members is addressed by several clauses in EN 1993-1-1.Primary interest of clauses in portal frames are

Uniform members in compression: This clause is primarily concerned with flexural buckling but also addresses torsional and torsional –flexural buckling (These lateral modes of failure will not govern the IPE sections and similar cross sections adopted for portal frames. This clause covers strut buckling resistance and the selection of buckling curves.

Uniform members in bending: This clause covers lateral –torsional buckling of beams. The distribution of bending moments along an unrestrained length of beam has an important influence on the buckling resistance. This is accounted for by the choice of C_1 factor when calculating M_{cr}

Uniform members in bending and axial compression: This clause addresses the interaction of axial load and moment in-plane and out-of-plane. The clause requires the following checks to be carried out unless full second order analysis, including all member imperfections$(P-\delta, torsional\ and\ lateral\ imperfections), is\ utilized.$

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}}+k_{yy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}}+k_{yz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}}\leq 1.0$$ ------Used to verify in-plane buckling--- Eq-4

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}}+k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}}+k_{zz}\frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}}\leq 1.0$$ ---Used to verify out-of-plane buckling.---Eq-5

For class 1,2,3 and bi-symmetric clss-4 sections $\Delta M_{y,Ed} = \Delta M_{z,Ed} = 0$ and $M_{z,Ed} -$ is zero because the frame is only loaded in its plane.

2.6.2 Restraints and member stability:

A) The need for restraints:-

i) Plastic hinges can form in the deep I-sections used

ii) Overall flexural buckling of the column and rafter about the minor axis does not occur.

iii) There is no lateral torsional buckling of an unrestrained compression flange on the inside of the member.

A restraint should be capable of resisting 2.5% of the compressive force in the members or part being restrained.

B) Column Stability:

The column contains a plastic hinge near the top at the bottom of the haunch. Below the hinge it is subjected to axial load and moment with the inside flange in compression. The code states in section 5.3.5(EN-1993-1-1) that torsional restraints (i.e., restraints to both flanges) must be provided at or within member depth D/2 from the plastic hinge.

In a member containing a plastic hinge the maximum distance from the restraint at the hinge to the adjacent restraint depends on whether or not restraint to the tension flange is taken into account. The following procedures apply.

i) Restraint to tension flange not taken into account:

This is the conservative method where the distance from the hinge restraint to the next restraint is given by

$$L_m = \frac{38\ r_y}{\left[\left(\frac{fc}{130} + \left(\frac{Py}{275}\right)^2 \left(\frac{x}{36}\right)^2\right)\right]^2} \text{ ---Eq-6}$$

$f_c -$ Compressive stress due to axial load.

$r_y -$ Radius of gyration about YY axis using the minimum value if the section varies.

$x -$ Torsional index using the maximum value if the section varies

When this method is used no further checks are required.

ii) Restraint to tension flange taken into account :

A method for determining spacing of lateral restraint taking account of restraint to the tension flange is given

$$L_t \leq \frac{620\, r_y\, x}{[72x^2 - 10^4]^{0.5}} \text{ --Eq-7}$$

C) Rafter stability near ridge:

The tension flange at the hinge in the rafter near the ridge is on the inside and no restraints are provided. The code BS in clause 5.5.3.1 specifying that a torsional restraint is not needed at the last hinge to form. In the portal two hinges form last near the ridge. A purlin is required at or near the hinge and purlins should be placed at a distance not exceeding L_m on each side of the hinge. In some cases a restraint to the inside flange is needed here when stresses reverse to compression under wind uplift loads.

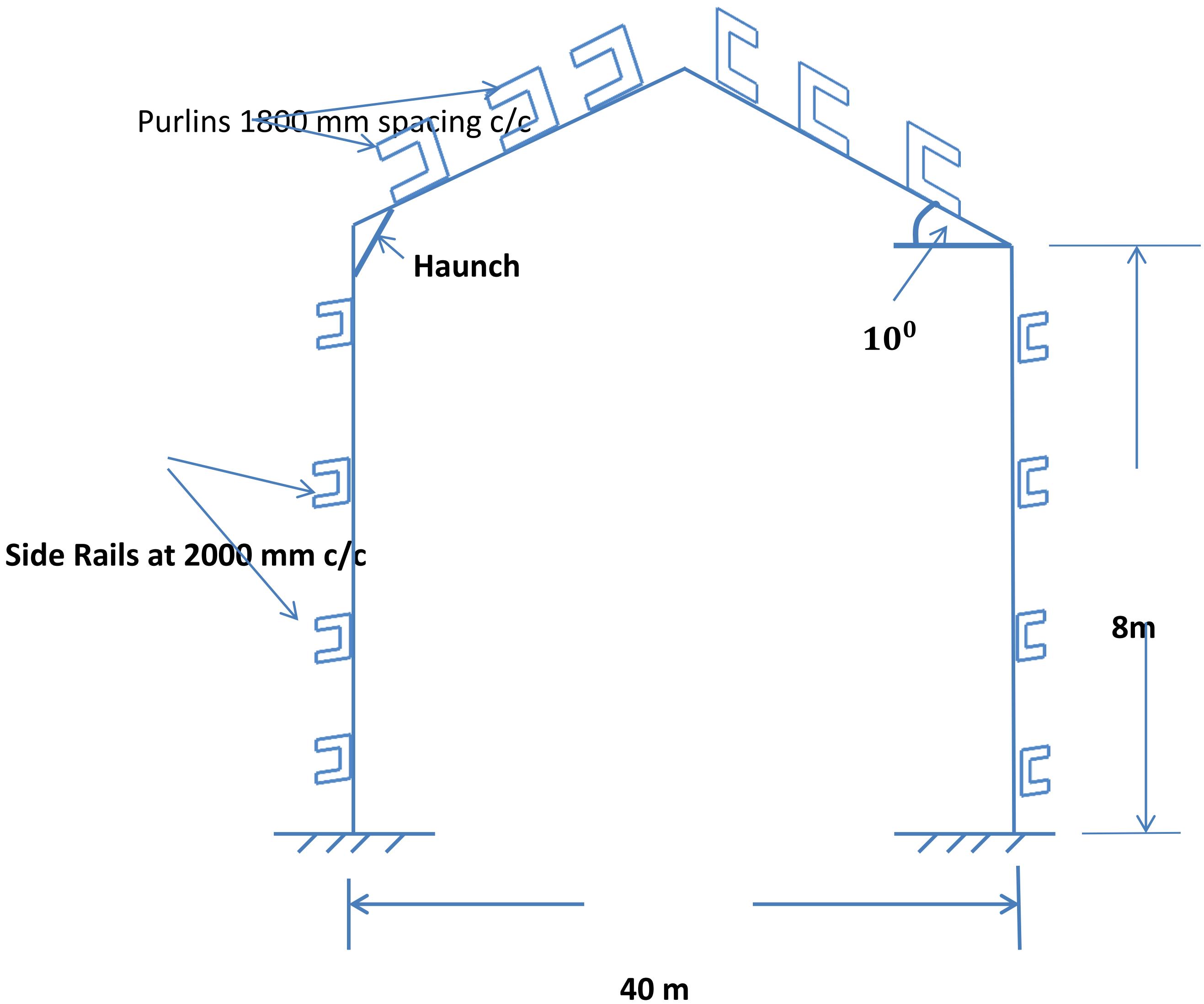
2.6.3 Out-of –plane restraint:

Three basic types of restraint that can be provided to reduce or prevent out-of-plane buckling.

i) Lateral Restraint:-which prevents lateral movement of the compression flange.

ii) Torsional restraint: - Which prevents rotation of a member about its longitudinal axis.

Intermediate Lateral restraint to the tension flange:-Such restraints are only of limited benefit, but do modify the out-of-plane buckling mode and may therefore allow the distance between torsional restraints to be increased



An Industrial Building located in Addis Ababa, Ethiopia, which is having doors and windows on all the four sides. The size of the door is 3m X 8m and the size of the window is 1.2 X 2m. On short wall side one door and two windows and on the longer wall side two doors and 8 windows are provided

Chapter 3

Analysis and Design

3.1 Wind Analysis:

For the industrial building of 40m wide and 75 long with a height of 8m at eaves with an angle of 10^0.The other dimensions are shown in the figure. The frame used is Gable frame and the cladding to the roof and walls is supported by purlins and rails. The side rails are provided at 2000 mm c/c and the purlins are located between 1500mm and 2000mm.

According to the EBCS-1995, the frame comes under category-H(roofs not accessible except for normal maintenance , repair , painting and minor repairs) from Table 2.3 categorization of roofs. And the location of the site is suburban or industrial areas plane ground. That is according to the EBCS-1995, the site location falling under category-III.

Step-i) Calculation of external and internal wind pressure

The other parameters for finding the wind pressure is temperature for Ethiopia is 20^0c and the site is located at an altitude of 1000m above M.S.L. Based upon the altitude the air density is going to change . Therefore for this altitude according to the EBCS-1995, Table 3.1 the air density $\rho = 1.06\, {}^{Kg}/_{m^3}$

The wind will create pressure on external and internal surfaces of the structure. This pressure on structure is calculated by

$W_e = q_{ref}c_e(z)c_{pe} \quad and \quad W_i = q_{ref}c_e(z)c_{pi}$ --- Eq-8

$q_{ref} -$ Reference mean wind velocity pressure $= \frac{\rho}{2}{V_{ref}}^2$ --Eq-9

$V_{ref} - C_{DIR}C_{TEM}C_{ALT}V_{ref,o}$ -- Eq-10

$V_{ref,o}$ $-$is the basic value of the reference wind velocity to be taken as 22 m/sec.

C_{DIR} $-$is the directional factor$= 1$

C_{TEM} $-$ is the temporary(or seasonal factor) to be taken as $= 1$

Therefore

$$V_{ref} = 22(1)(1)(1) = 22\ m/sec$$

Step-ii). Finding terrain and roughness coefficient

$c_e(z) = {C_r}^2(z){C_t}^2(z)\left[\frac{7K_T}{C_r^{\cdot}(z)C_t^{\cdot}(z)}\right]$ -- EQ-11

K_T −Terrian factor

$C_r^{\cdot}(z)$ − is the roughness coefficient

$C_t^{\cdot}(z)$ − Topography coefficient.

For the plane ground the slope $\emptyset < 0.05$, therefore $C_t = 1$

And the site is located in the suburban and industrial area , which falls under category-III, the structure total height is 11.52m , the relevant values from Table-3.2, Terrain categories and Related parameters. $K_T = 0.22, \quad z_o(m) = 0.3\ and\ \ z_{min}(m) = 8$

Therefore for $8 \leq 11.52 \leq 200$

$C_r(z) = K_T ln\left(\frac{z}{z_0}\right) = \ 0.22\ ln\left(\frac{11.52}{0.3}\right) = 0.80$ --- Eq-12

From the above values the value for $c_e(z) = 0.8^2\ 1^2\left[1 + \frac{7K_T}{08*1}\right] = 1.868$

Step-iii) Finding pressure coefficients for walls

External Pressure Coefficients for walls:

The pressure coefficient on wall depends up on d $- the\ width\ of\ the\ structure, here\ it\ is\ 40m$ and $e\ value(e = b\ or\ 2h - minimum)$

$$e = 75m\ or\ 2 * 11.52 = 23.04\ m$$

The case $d > e \quad i.e, 40 > 23.04m$

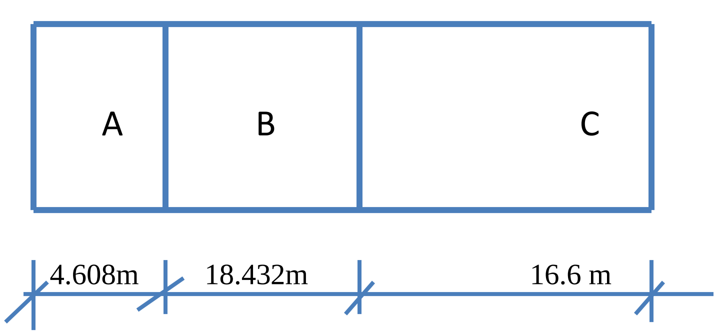


Fig.3.1 Division wall areas according to code EBCS-1, 1995.

From Table A-1 and for $\frac{d}{h} > 4$

Areas of A, B and C are greater than $10m^2$

A	B	C	D	E
$Area > 10m^2$	$Area > 10m^2$	$Area > 10m^2$	$Area > 10m^2$	$Area > 10m^2$
$Cpe_{10} = -1.0$	$Cpe_{10} = -0.8$	$Cpe_{10} = -0.5$	$Cpe_{10} = +0.6$	$Cpe_{10} = -0.3$

Table.3.1 External Pressure coefficients based on area

Step-iv) Finding pressure coefficients for gable roof

For Gable Roof:

In the case of duo pitch roofs the external pressure coefficients are depends upon the direction of the wind . If the wind is parallel to the shorter walls called $\theta = 0^0$ and if the wind perpendicular to the shorter walls it is considered the case as $\theta = 90^0$.

Case-1: Wind is parallel to the shorter walls. i.e, $\theta = 0^0$

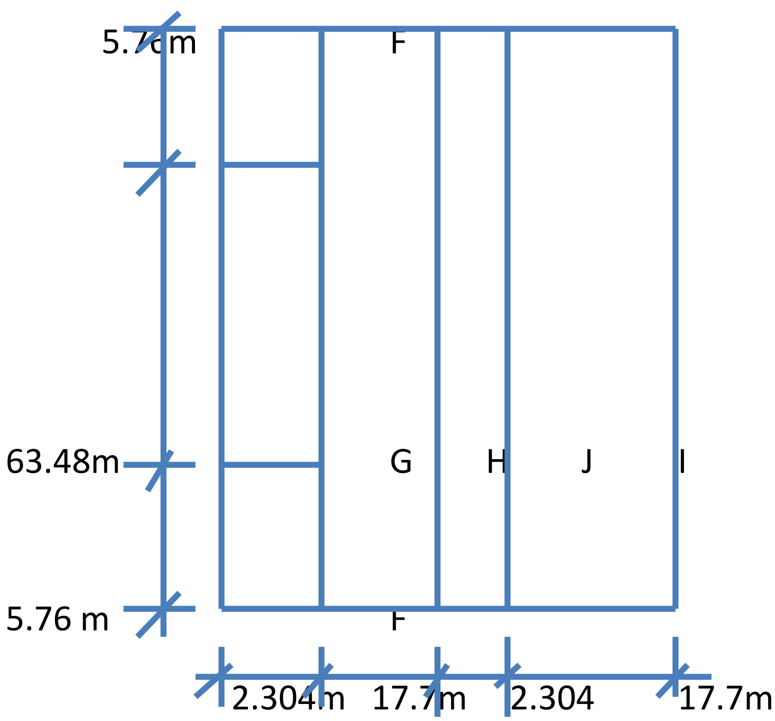


Fig.3.2 Plan division into areas according to the code EBCS-1,1995

External pressure coefficients: for$\boldsymbol{\theta = 0^0}$.

Location	F	G	H	I	J
Area of locations	Area $> 10\mathrm{m}^2$	Area $> 10\mathrm{m}^2$	Area $> 10\mathrm{m}^2$	Area $> 10\mathrm{m}^2$	Area $> 10\mathrm{m}^2$
For $5^0 \mathrm{Cpe}_{10}$	-1.7	-1.2	-0.6	-0.3	-0.3
For 15^0 Cpe_{10}	-0.9/+0.2	-0.8/+0.2	-0.3/+0.2	-0.4	-1.0
For 10^0 max.v	-0.75	-1.0	-0.45	-0.35	-0.65

Table.3.2. External pressure coefficients for plan areas for $\boldsymbol{\theta = 0^0}$.

Similarly external pressure coefficients for $\boldsymbol{\theta = 90^0}$.

Location	F	G	H	I
Area of locations	Area $> 10\mathrm{m}^2$	Area $> 10\mathrm{m}^2$	Area $> 10\mathrm{m}^2$	Area $> 10\mathrm{m}^2$
For $5^0 \mathrm{Cpe}_{10}$	-1.6	-1.3	-0.7	-0.5
For 15^0 Cpe_{10}	-1.3	--1.3	-0.6	-0.5
For 10^0 max.v	-1.45	--1.3	-0.5	-0.5

Table.3.3. External pressure coefficients for plan areas for $\boldsymbol{\theta = 90^0}$.

Step-V) Finding internal pressure coefficients

Internal Pressure Coefficient $\mathbf{C_{pi}}$:

For finding internal pressure coefficient for a building without internal partitions from figure A.11, the graph is a function of opening ratioμ value.

$$\mu = \frac{\text{Area od openings at the leewardand wind parallel sides.,}}{\text{area of openings at the windward,leeward and wind paralle sides.}} \text{–Eq-13}$$

The structure is having doors and windows on all the four sides. The size of the door is 3m X 8m and the size of the window is 1.2 X 2m. On short wall side one door and two windows and on the longer wall side two doors and 8 windows.

$$\mu = \frac{163.2}{192} = 0.85$$

For $\mu = 0.85$, rom figure A-11, the value for $C_{pi} = -0.45$

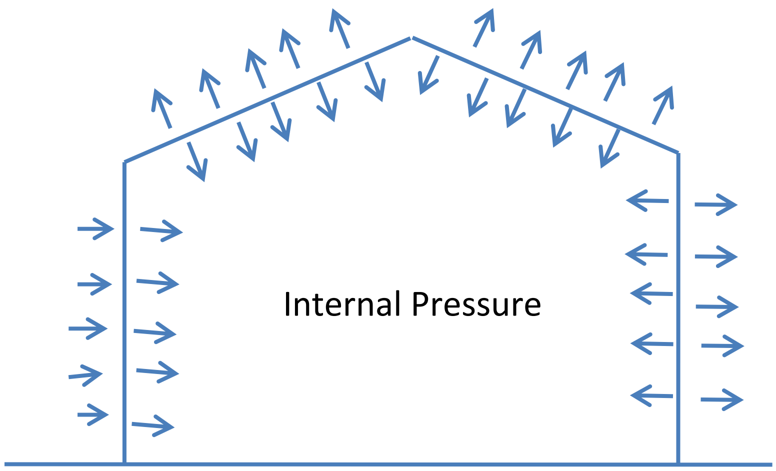


Fig 3.3 Pressure directions on surfaces

Step-Vi) Finding pressure on roof and wall

Pressure on Roof:

For the external pressure, the external pressure coefficients with the wind direction perpendicular to the shorter walls is more i.e, for $\theta = 90^0$

Location	F	G	H	I
Internal pressure	-18.538(kg/m2)	-18.538	-18.538	-18.538
External pressure	-59.6(kg/m2)	-53.4	-20.5	-20.5

Table.3.4 Maximum Pressures in plan areas

Pressure on walls:

Location	A	B	C	D	E
Internal pressure	-18.538(kg/m2)	-18.538	-18.538	-18.538	-18.538
External pressure	-41.09(kg/m2)	-32.8	-20.548	+24.65	-12.32

Table.3.5 Maximum pressures on wall areas

In BS 6369: part-2, for all structures less than 100m in height and where the wind loading can be represented by equivalent static loads , the wind loading can be obtained by using one or a combination of the following methods.

i) Standard method uses a simplified procedure to obtain a standard effective wind speed, which is used with standard pressure coefficients to determine the wind loads

ii) Directional method in which effective wind speeds and pressure coefficients are determined to derive the wind loads for each wind direction.

The wind loads should be calculated for each of the loaded areas under consideration , depending on the dimensions of the building . These may be

i) The structure as a whole.

ii) Parts of the structure , such as walls and roofs or

iii) Individual structural components , including cladding units and their fixings.

3.2 Elastic Analysis:

Loads:

Permanent loads

G = Gself-weight + Groof

Gself-weight: self-weight of the beams

Groof: roofing with purlins Groof = 0.30 kN/m2

For an internal frame: Groof = 0.30 × 7.50 = 2.25 kN/m

Imposed load on roof

Characteristic values for loading on the roof (type H: not accessible).

From Table 2.14 Imposed loads on roofs from EBCS-1,1995 qk = 0.25 kN/m2

For an internal frame: qk = 0.25 × 7.50 = 1.875kN/m

Load combinations

Therefore, the critical design combination for choosing the member size is:

Where:

$\gamma_p = 1.3$ (Permanent actions)

$\gamma_v = 1.6$ (Variable actions)

Preliminary sizing:

Single-story steel buildings. Part 2: Concept design provides a table of preliminary member sizes, according to the rafter load and the height to eaves.

Rafter load = 1.3(2.25+ self-weight) +1.6 (1.875) +4.47 = 10.395 kN/m + self-weight

Say 11 kN/m to include self-weight.

The section chosen for the rafter is an IPE 600, S355

The section chosen for the column is an IPE650, S355

Buckling amplification factorα_{cr}:

In order to evaluate the sensitivity of the frame to 2nd order effects, the buckling amplification factor, cr, has to be calculated. This calculation requires the deflections of the frame to be known under a given load combination.

An elastic analysis is performed to calculate the reactions under vertical loads at ULS, which provides the following information:

The vertical reaction at each base $V_{Ed} = 168$ KN, The horizontal reaction at each base $H_{Ed} = 116$ KN. And the maximum axial force in rafters $N_{R,Ed} = 130$KN

Axial compression in the rafter:

According to the code, if the axial compression in the rafter is significant then cr is not applicable. In such situations, Appendix B of the Euro Code document recommends the use of $\alpha_{cr,rest}$ instead.

The axial compression is significant if $\bar{\lambda} \geq 0.3 \sqrt{\frac{Af_y}{N_{Ed}}}$ or if $N_{Ed} \geq 0.09\ N_{cr}$, which is an equivalent expression.

NEd is the design axial load at ULS in the rafter

Lcr is the developed length of the rafter pair from column to column.

$$L_{cr} = \frac{40}{\cos 10^0} = 40.46\text{m}$$

$N_{cr} = \frac{\pi^2 EI_z}{L_{cr}{}^2} = \frac{\pi^2 X210000x33740X10^4}{(40.46X10^3)^2} 10^{-3} = 427.27$ kn ---Eq-14

Therefore 0.09 $N_{cr} = 0.09X427.27$=38.45kn

NR,Ed = 130 KN > 38.45KN

Therefore the axial compression in the rafter is insignificant and α_{cr} from EN 1993-1-1 is not applicable

Calculation of $\alpha_{cr,est}$

For a pitched roof frame: $\alpha_{cr,est} = \min(\alpha_{cr,s,est}; \alpha_{cr,r,est})$

$\alpha_{cr,r,est}$ only needs to be checked for portal frames of 3 or more spans.

When assessing frame stability, allowance can be made for the stiffness. In this example, a base stiffness equal to 10% of the column has been assumed to allow for the nominally pinned bases. To calculate cr, a notional horizontal force is applied to the frame and the horizontal deflection of the top of the columns is determined under this load.

The notional horizontal force is:

$$H_{NHF} = \frac{1}{200} V_{Ed} = \frac{1}{200} 168 = 0.84 \text{ KN}$$

The horizontal deflection of the top of the column under this force is obtained from the elastic analysis as 1.6 mm.

$\alpha_{cr,s,est} = 0.8 \left\{1 - \left(\frac{N_{R,Ed}}{N_{R,cr}}\right)\right\} \left\{\frac{1}{200} \frac{h}{\delta_{NHF}}\right\}$---Eq-15

$$= 0.8 \left\{1 - \frac{130}{427.27}\right\} \left\{\frac{1}{200} \frac{8000}{1.6}\right\} = 13.9$$

$$\alpha_{cr,est} = \alpha_{cr,s,est} = 13.9 > 10$$

Therefore first order elastic analysis may be used and second order effects do not to be allowed for.

Frame Imperfections:

The global initial sway imperfections may be determined from

$$\emptyset = \emptyset_0 \alpha_h \alpha_m$$

Where $\emptyset_0 = \frac{1}{200}$

$\alpha_h = \frac{2}{\sqrt{h}} = \frac{2}{\sqrt{8}} = 0.707$

$\alpha_m = \sqrt{0.5\left(1+\frac{1}{m}\right)}$= 0.87, Where m= 2(number of columns.

Therefore $\emptyset = \frac{1}{200}X0.707X0.87 = 3.07X10^{-3}$

Initial sway imperfections may be considered in two ways:

i) By modeling the frame out of plumb
ii) By applying equivalent horizontal forces(EHF)

Applying equivalent horizontal forces is the preferred option and the method that is used in this problem. The equivalent horizontal forces are calculated as

$H_{EHF} = \emptyset V_{Ed}$

However sway imperfections may be disregarded where $H_{Ed} \geq 0.15V_{Ed}$

Below table shows the total reactions for the structure to determine H_{Ed} and V_{Ed}

	Left column		Right Column		Total Reaction		0.15 V_{Ed}
	H_{Ed}	V_{Ed}	H_{Ed}	V_{Ed}	H_{Ed}	V_{Ed}	
Reactions	116	168	-116	168	0	336	50

Table.3.6 Forces in horizontal and vertical directions

$$H_{Ed} = 0 < 0.15V_{Ed}$$

Therefore the initial sway imperfections have to be taken into account.

The equivalent horizontal forces:

$H_{EHF} = \emptyset V_{Ed} = 3.07X10^{-3}X168 = 0.51kn$, this force is greater than wind force

The force is applied at the top of each column, in combination with the permanent and variable actions. For the Ultimate limit state analysis, the bases are modeled as pinned. Otherwise the base details and foundation would need to be designed for the resulting moment.

Member verification:

The cross section verification and the buckling resistance are verified for each member.

Cross section verification

The resistance of the cross section is to done form the perspective of the shear resistance, Compression resistance and Bending moment resistance. In addition, bending and shear interaction, as well as bending and axial force interaction must be verified.

They have to satisfy i.e, $V_{Ed} \leq V_{pl,Rd}$, $N_{Ed} \leq N_{c,Rd}$, and $M_{Ed} \leq M_{pl,y,Rd}$

Buckling Verification:

The rafters and columns must be verified for out-of –plane buckling between restraints and in plane buckling

The buckling checks due to the interaction of axial force and bending moment are carried out

$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1.0$------Used to verify in-plane buckling Eq-16

$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed}+\Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1.0$---Used to verify out-of-plane buckling. Eq-17

For single story frames these expressions can be simplified as follows.

$\Delta M_{y,Ed} = 0 \; and \; \Delta M_{z,Ed} = 0 \; for$ Class-1, class-2, and class-3 sections $M_{z,Ed} = 0$

Column Section: IPE500 and $f_y = 355 \, {}^{N}/_{mm^2}$

Cross section classification---

The web is falling under clas-1 and the flange is also falling under class-1, therefore the total section is class-1. The verification of the member will be based on the plastic resistance of the cross section

Elastic analysis is the most common method of analysis for general structures, but will usually give less economical portal structures than plastic analysis. EN 1993-1-1 allows the plastic cross-sectional resistance to be used with the results of elastic analysis, provided the section class is Class 1 or Class 2. In addition, it allows 15% of moment redistribution as defined in EN 1993-1-1 5.4.1.4(B)

Shear resistance:

$$shear\ area\quad A_v = A - 2bt_f + (t_w + 2r)t_f\ not\ less\ than\quad \eta h_w t_w$$

Therefore $V_{Ed} = 117KN < 1250KN$

Bending and shear interaction:

When shear force and bending moment act simultaneously on a cross section, the shear force can be ignored it is smaller than 50% of the plastic shear resistance

$$V_{Ed} = 117KN < 0.5V_{pl,rd} = 0.5X1250 = 625kn$$

Therefore the effect of the shear force on the bending moment resistance may be neglected.

Compression resistance

$$N_{c,Rd} = \frac{Af_y}{\gamma_{Mo}} = 4118\ KN$$

Therefore $N_{Ed} = 168kn\ \leq N_{C,Rd}$

Bending and axial force interaction:

When axial force and bending moment act simultaneously on a cross section, the axial force can be ignored provided the following two conditions are satisfied.

i) $V_{Ed} \leq 0.25N_{pl,Rd}\ and \quad ii)\ V_{Ed} \leq \frac{0.5h_w t_w f_y}{\gamma_{Mo}} = \frac{0.5X468X10.2355}{1.0} = 847kn$

Therefore 168KN<1030kn and 168Kn< 847kn

Bending moment resistance:

$$M_{pl,y,Rd} = \frac{2194X10^3X355}{1.0} = 779kn$$

Therefore 669 <779 kn

Out-of plane of buckling:

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$ Eq-18

This expression should verified between torsional restraints.

In order to take the advantage of the restraints between torsional restraints on tension flange , spacing between the restraints to the tension flange is small enough.

In order to determine whether or not the spacing between restraints is small enough, Annex BB of EN-1993-1-1 provides an expression to calculate the maximum spacing .

Verification of spacing between intermediate restraints:

In this case the restraint to the tension flange is provided by the side rails .These side rails are spaced at 1900mm

Therefore the limiting spacing is

$L_m = \frac{38i_z}{\sqrt{\frac{1}{57.4}\left(\frac{N_{Ed}}{A}\right)+\frac{1}{756C_1{}^2}\frac{W_{pl,y}{}^2}{AI_t}\left(\frac{f_y}{235}\right)^2}}$ Eq-19

C_1 −is a factor that account for the shape of the bending moment diagram. For linear bending moment diagram C_1 −depends on the ratio of the minimum and maximum bending moments in the segment being considered.

The ratios of the bending moments for the middle and bottom segments of the column (without considering the haunch) are as follows.

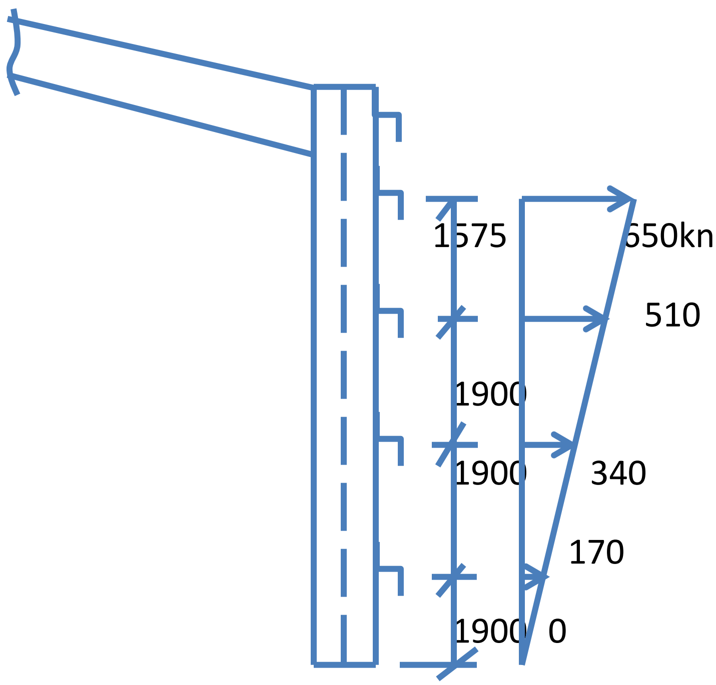


Fig.3.4.Variation of moment along column at side rails position.

$\psi = \frac{340}{510} = 0.67$ for this the $C_1 = 1.24$

$\psi = \frac{170}{340} = 0.5$ for this the $C_1 = 1.31$

$\psi = \frac{0}{170} = 0$ for this the $C_1 = 1.77$

$C_1 = 1.31$ is the most onerous case and therefore this is the case that will be analyzed

Therefore $L_m = 1584mm < side\ rail\ spacing\ \ 1900mm$

Therefore normal design procedure must be adopted and advantage may not be taken of the restraint to the tension flange.

Whole column verification (7275mm)

If the flexural buckling and, lateral torsional buckling and interaction checks are satisfied for the length of the whole column, no further restraints are required. Otherwise, intermediate torsion restraints will be introduced to the column, or the column size is increased.

$$\frac{h}{b} = \frac{500}{200} = 2.5$$

$$t_f = 16mm$$

Buckling about Z-z axis: Curve "b" for hot rolled "I" sections, $\alpha_z = 0.34$

Therefore $\lambda_1 = 76.4$

$\bar{\lambda}_z = 1.60$ and $\phi_z = 0.5[1 + 0.34(1.60 - 0.2) + 1.60^2] = 2.02$

$\chi_z = 0.307$

$N_{b,z,Rd} = \frac{\chi_z A f_y}{\gamma_{M1}}$ -- Eq-20

$$= \frac{0.307X11600X355}{1.0} 10^{-3} = 1264kn$$

$N_{Ed} = 168kn < 1264kn$ OK

Lateral Torsional Buckling resistance$M_{b,Rd}$:

The lateral torsional buckling resistance of member is calculated as a reduction factor χ_{LT} multiplied by the section modulus and the yield strength of the section .The reduction factor is calculated as a function of the slenderness$\bar{\lambda}_{LT}$, which depends on the critical moment of the member. The expression for the critical moment M_{cr} is given below. The factor C_1 accounts for the shape of bending moment diagram of the member. For the case of a linear bending moment diagram, C_1 depends on the ratio of the bending moments at the ends of the member, given as ψ

For the total length of the column (without the haunch):

$$\psi = \frac{0}{616} = 0 \quad for\ this\ value\ C_1 = 1.77$$

$M_{cr} = C_1 \frac{\pi^2 E I_Z}{L^2} \sqrt{\frac{I_W}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z}}$ -- Eq-21

$$M_{cr} = 1.77\frac{\pi^2 X210000X2142X10^4}{7275^2}X\sqrt{\frac{1249X10^9}{2142X10^4}+\frac{7275^2X81000X89.3X10^4}{\pi^2X210000X2124X10^4}}$$

$$= 561X10^6\, n - mm$$

The non-dimensional slenderness $\bar{\lambda}_{LT}$ is calculated as

$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{2194X10^3X355}{561X10^6}} = 1.178$ Eq-22

For the calculation of the reduction factorχ_{LT}, EN 1993-1-1 provides two methods but the general method, applicable to any section, is given in clause 6.3.2.2 and 6.3.2.3 that can only be used for rolled sections or equivalent welded sections

In this case the second method is used i.e., given in clause 6.3.2.3

$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta\bar{\lambda}_{LT}{}^2\right]$ –Eq-23

En 1993-1-1 recommends the following values:

$$\bar{\lambda}_{LT,o} = 0.4\ and\ \beta = 0.75$$

$For\ \frac{h}{b} = 2.5$, curve “c” for hot rolled I $sections$, Therefore $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5[1 + 0.49(1.17 - 0.4) + 0.75(1.17)^2] = 1.202$$

$\chi_{LT} = \frac{1}{\phi_{LT}+\sqrt{\phi_{LT}{}^2-\beta\bar{\lambda}_{LT}{}^2}} = \frac{1}{1.202+\sqrt{1.202^2-0.75X1.17^2}} = 0.54$ Eq-24

Therefore $M_{b,Rd} = \frac{\chi_{LT}W_{pl,y}f_y}{\gamma_{M1}} = \frac{0.54X2194X10^3X355}{1.0}X10^{-6} = 421\ kn - m$ Eq-25

$$M_{b,Rd} = 421 < 650\ kn - m$$

Since the check for lateral torsional buckling resistance alone fails , the interaction of axial force and bending moment is not carried out.

It is necessary to introduce a torsional restraint between the haunch and the base, as shown below. The bending moment is greater at the top of the column and therefore the restraint is placed closer to the maximum bending moment , rather than in the middle of the column.

The restraint must be provided at side rail position , sine the bracing from the side rail to the inner flange is used to provide the torsional restraint.

Upper segment: (1575mm)

As previously, the flexural buckling and lateral torsional buckling checks are carried out separately before proceeding to verify the interaction between the two.

Flexural buckling resistance about the minor axis, $N_{b,z,Rd}$:

$$\frac{h}{b} = \frac{500}{200} = 2.5, \quad t_f = 16mm$$

Buckling about z-z axis,

Curve “b” for hot rolled “I” sections., the imperfection factor is $\alpha = 0.34$

$$\lambda_1 = 76.4$$

$\bar{\lambda}_z = \frac{L_{cr}}{i_z}\frac{1}{\lambda_1} = \frac{1575}{43.1}\frac{1}{76.4} = 0.48$ --Eq-26

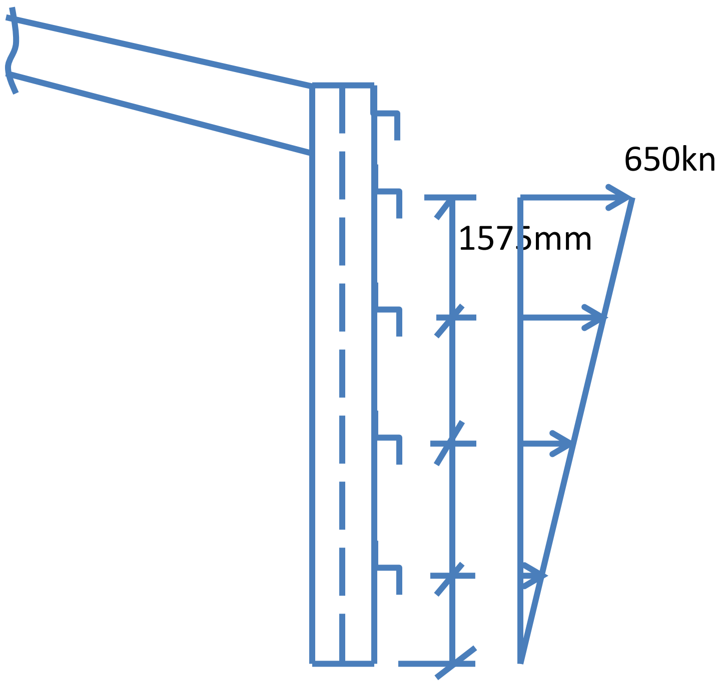


Fig.3.5 Maximum moment at bottom of haunch

$$\phi_z = 0.5[1 + 0.34(0.48 - 0.2) + 0.48^2] = 0.66$$

$$\chi_z = \frac{1}{\phi_z + \sqrt{{\phi_z}^2 - {\bar{\lambda}_z}^2}} = \frac{1}{0.66 + \sqrt{0.66^2 - 0.48^2}} = 0.89$$

$$N_{b,z,Rd} = \frac{\chi_z A f_y}{\gamma_{M1}} = \frac{0.89X11600X355}{1.0} X10^{-3} = 3665kn$$

$N_{Ed} = 168kn < 3665kn$ OK

Lateral Torsional Buckling resistance, $M_{b,rd}$

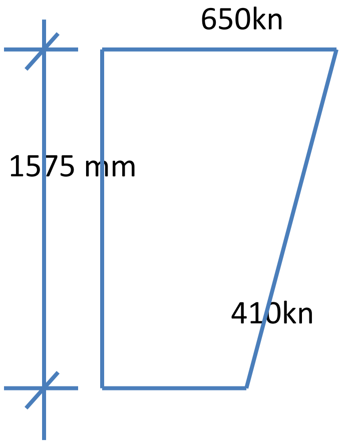


Fig.3.6 Moment variation between the first side rail at haunch and next to it

$$\psi = \frac{410}{650} = 0.63, \quad for\ this\ the\ C_1 = 1.26$$

$$M_{cr} = 1.26\frac{\pi^2 X210000X2142X10^4}{1575^2}X\sqrt{\frac{1249X10^9}{2142X10^4} + \frac{1575^2X81000X89.3X10^4}{\pi^2X210000X2124X10^4}}$$

$$= 5632kn - m$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{2194X10^3X355}{5632X10^6}} = 0.371$$

For hot rolled sections

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta\bar{\lambda}_{LT}{}^2\right]$$

$$\phi_{LT} = 0.5[1 + 0.49(0.371 - 0.4) + 0.75(0.371^2)] = 0.544$$

$$\chi_{LT} = \frac{1}{0.544 + \sqrt{0.544^2 - (0.75)0.371^2}} = 1.017 > 1.0$$

$$Therefore\ \ \chi_{LT} = 1.0$$

$$M_{b,Rd} = \frac{\chi_{LT} W_{pl,y} f_y}{\gamma_{M1}} = 779\ kn - m$$

$M_{Ed} = 650 < 779\ kn - m$ OK

Interaction of axial force and bending moment:

Out-of-plane Buckling due to the interaction of axial force and bending moment is verified by satisfying the following expression.

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$ Eq-28

$\bar{\lambda}_z \geq 0.4$, the interaction factor ,k_{zy} is calculated as

$k_{zy} = max\left[\left(1 - \frac{0.1\bar{\lambda}_z}{C_{mLT}-0.25}\frac{N_{Ed}}{N_{b,Rd,z}}\right); \left(1 - \frac{0.1}{C_{mLT}-0.25}\frac{N_{Ed}}{N_{b,Rd,z}}\right)\right]$ --Eq-29

$C_{mLT} = 0.6 + 0.4\psi$ ---Eq-30

$$\psi = \frac{410}{650} = 0.63$$

$$C_{mLT} = 0.6 + 0.4X0.63 = 0.85 > 0.4$$

Therefore $C_{mLT} = 0.85$

$$k_{zy} = max\left[\left(1 - \frac{0.1X0.48}{0.85 - 0.25}\frac{168}{3665}\right); \left(1 - \frac{0.1}{0.85 - 0.25}\frac{168}{3665}\right)\right]$$

$= max[0.996; 0.92]$=0.996

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{168}{3665} + 0.996\frac{650}{779} = 0.87 < 1.0$ OK

Lower segment (5700mm):

As previously the flexural buckling resistance and the lateral torsional buckling resistance are checked individually and then the interaction between the two is verified

Flexural buckling resistance about the minor axis, $N_{b,Z,Rd}$:

Curve “b” for hot rolled “I” Sections

$$\alpha_z = 0.34$$

$$\lambda_1 = 76.4$$

$$\bar{\lambda}_z = \frac{L_{cr}}{i_z}\frac{1}{\lambda_1} = \frac{5700}{43.1}\frac{1}{76.4} = 1.73$$

$$\phi_z = 0.5[1 + 0.34(1.73 - 0.2) + 1.73^2] = 2.22$$

$$\chi_z = \frac{1}{2.22 + \sqrt{2.22^2 - 1.73^2}} = 0.28$$

$$N_{b,Z,Rd} = \frac{0.28X11600X355}{1.0}X10^{-3} = 1153KN$$

$$N_{Ed} = 168kn < 1153kn \qquad OK$$

Lateral Torsional Buckling Resistance, $M_{b,Rd}$:

As previously the C_1 factor needs to be calculated in order to determine the critical moment of the member.

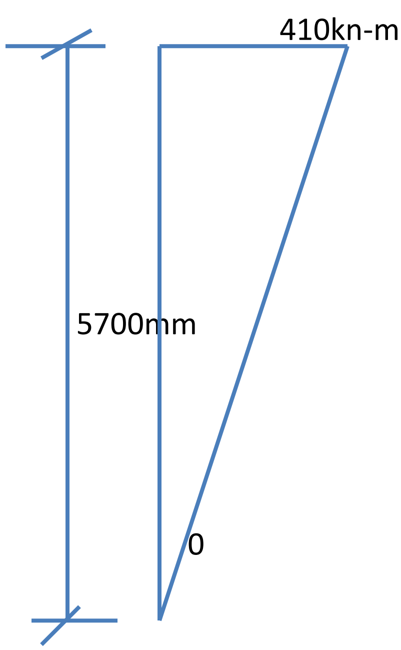


Fig.3.7. Moment variation at bottom of haunch and bottom of the column

$$\psi = \frac{0}{410} = 0 \;\; , for\ this\ value\ \ the\ C_1 = 1.77$$

$$M_{cr} = 1.77\frac{\pi^2 X210000X2142X10^4}{5700^2}X\sqrt{\frac{1249X10^9}{2142X10^4} + \frac{5700^2X81000X89.3X10^4}{\pi^2X210000X2124X10^4}}$$

$$= 808.3kn - m$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{2194X10^3X355}{808.3X10^6}} = 0.98$$

Curve “c” for hot rolled “I” sections: Therefore $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5[1 + 0.49(0.98 - 0.4) + 0.75(0.98^2)] = 1.00$$

$$\chi_{LT} = \frac{1}{1. + \sqrt{1^2 - 0.75(0.98^2)}} = 0.65$$

$$M_{b,Rd} = \frac{0.65X2194X10^3X355}{1.0} = 506kn - m$$

$$M_{Ed} = 410kn - m < 506kn - m \qquad ok$$

Interaction of axial force and bending moment:

Out-of-plane buckling due to the interaction of axial force and bending moment is verified by satisfying the following expression

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$ Eq-31

$\bar{\lambda}_z \geq 0.4$, the interaction factor,k_{zy} is calculated as

$k_{zy} = max\left[\left(1 - \frac{0.1\bar{\lambda}_z}{C_{mLT} - 0.25}\frac{N_{Ed}}{N_{b,Rd,z}}\right); \left(1 - \frac{0.1}{C_{mLT} - 0.25}\frac{N_{Ed}}{N_{b,Rd,z}}\right)\right]$ Eq-32

$$C_{mLT} = 0.6 + 0.4\psi$$

$$\psi = \frac{0}{410} = 0$$

$$C_{mLT} = 0.6 + 0.4X0 = 0.6 > 0.4$$

Therefore $C_{mLT} = 0.6$

$$k_{zy} = max\left[\left(1 - \frac{0.1X1.73}{0.6 - 0.25}\frac{168}{1153}\right);\ \left(1 - \frac{0.1}{0.6 - 0.25}\frac{168}{1153}\right)\right]$$

$= max[0.927; 0.95]$=0.95

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{168}{1153} + 0.95\frac{410}{506} = 0.914 < 1.0$ OK

In − plane Buckling :

The in plane buckling interaction is verified with the following expression

$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$ Eq-33

The maximum design values of either column occur on the right hand column (considering EHF applied from left to right) and are as follows.

$M_{Ed} = 650kn - m$

$N_{Ed} = 168kn$

Firstly individual checks are carried out for flexural buckling alone and lateral torsional buckling alone. Then the interaction expression for in-plane buckling is applied to verify that the combination of axial force and bending moment does not cause excessive buckling on the columns.

Flexural buckling resistance about the major axis, $N_{b,y,Rd}$:

$$\frac{h}{b} = \frac{500}{200} = 2.5$$

$$t_f = 16mm$$

Buckling is about y-y axis.

Curve "a" for hot rolled "I" Sections

$$\alpha_y = 0.21$$

The buckling length is the system length, which is the distance between nodes (i.e, the length of the column, L=8000m

$$\lambda_1 = 76.4$$

$$\bar{\lambda}_z = \frac{L_{cr}}{i_y}\frac{1}{\lambda_1} = \frac{8000}{204}\frac{1}{76.4} = 0.51$$

$$\phi_z = 0.5[1 + 0.21(0.51 - 0.2) + 0.51^2] = 0.66$$

$$\chi_z = \frac{1}{0.66 + \sqrt{0.66^2 - 0.51^2}} = 0.92$$

$$N_{b,Z,Rd} = \frac{0.92X11600X355}{1.0}X10^{-3} = 3816KN$$

$$N_{Ed} = 168kn < 3816\ kn \qquad OK$$

Lateral Torsional Buckling Resistance,$M_{b,Rd}$:

$M_{b,Rd} -$ is the least buckling moment resistance of those calculated previously

$$M_{b,Rd} = \min(779; 506)$$

Therefore $M_{b,Rd} = 506\ kn - m$

Interaction of axial force and bending moment:

In-Plane buckling due to the interaction of axial force and bending moment is verified by satisfying the following expression.

$$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

For $C_{my,}$ the relevant braced points are the torsional restraints at the end of the member

the interaction factor ,k_{yy} is calculated as follows.

$$k_{yy} = min\left[C_{my}\left(1 + (\bar{\lambda}_y - 0.2)\frac{N_{Ed}}{N_{b,y,Rd}}\right); C_{my}\left(1 + 0.8\frac{N_{Ed}}{N_{b,y,Rd}}\right)\right] \quad \text{EQ-34}$$

$$C_{my} = 0.6 + 0.4\psi > 0.4$$

$$\psi = 0$$

$$C_{mLT} = 0.6 + 0.4X0 = 0.6 > 0.4$$

Therefore $C_{mLT} = 0.6$

$$k_{zy} = min\left[0.6\left(1 + (0.385 - 0.2)\frac{168}{3816}\right); 0.6\left(1 + 0.8\frac{168}{3816}\right)\right]$$

$= min[0.604; 0.62]$=0.604

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{168}{3816} + 0.604\frac{650}{506} = 0.82 < 1.0$ OK

Validity of the column section

i) It has been demonstrated that the cross sectional resistance of the section is greater than the applied forces.

ii) The Out-of-Plane and in-plane buckling checks have been verified for the appropriate choice of restraints along the column.

iii) Therefore it is concluded that the IPE 650 section S355 steel is appropriate for use as columns in this portal frame.

Rafter:

IPE600,S355

$$V_{Ed} = 132kn(max.val)$$

$$N_{Ed} = 141kn(max.val)$$

$$M_{Ed} = 374kn(\max val)$$

Section properties: A=9880mm^2 $t_w = 9.4mm$, $t_f = 14.6mm$, $W_{pl,y} = 1702X10^3$ mm^3

Cross section classification:

Web- From Table 5.2 of EBCS, 2013.

Part subjected bending and compression, check is depends on the α value

$\alpha = \frac{d_W + d_N}{2d_W}$

Where $d_N = \frac{N_{Ed}}{t_W f_y} = \frac{141X10^3}{9.4X355} = 42.7$ Eq-35

d_W −depth of the web portion =378.8mm

$$\alpha = \frac{378.8 + 42.7}{2X378.8} = 0.556 > 0.5$$

If $\alpha > 0.5$ $from\ the\ Table$ 5.2 $the\ limit\ for\ class - 1\ is$ $C/_t \leq \frac{396\varepsilon}{13\alpha - 1} = \frac{396X0.81}{13(0.556) - 1} =$ 52.1

$$C/_{t_W} = \frac{378.8}{9.4} = 40.3 < 52.1$$

Therefore the web is class-1

The flange:

$$C/_{t_f} = \frac{69.3}{14.6} = 4.7$$

The limit class-1 is: $9\varepsilon = 9X0.81 = 7.3$

Therefore $4.7 < 7.3$, The flange is class-1

Therefore the section is class-1, the verification of the section is based on the plastic resistance of the cross section

Shear resistance:

$shear\ area\ \ A_v = A - 2bt_f + (t_w + 2r)t_f\ not\ less\ than\ \ \eta h_w t_w$ Eq-36

$$A_v = 5082mm^2 > 1.0X420.8X9.4 = 3956\ mm^2$$

$V_{Pl,Rd} = \frac{A_v(\frac{f_y}{\sqrt{3}})}{\gamma_{Mo}} = \frac{5082\left(\frac{355}{\sqrt{3}}\right)}{1.0} = 1042Kn$ Eq-36

Therefore $V_{Ed} = 132KN < 1042kn$

Bending and shear interaction:

When shear force and bending moment act simultaneously on a cross section, the shear force can be ignored it is smaller than 50% of the plastic shear resistance

$$V_{Ed} = 132KN < 0.5V_{pl,rd} = 0.5X1042 = 521kn \qquad OK$$

Therefore the effect of the shear force on the bending moment resistance may be neglected.

Compression resistance

$$N_{c,Rd} = \frac{Af_y}{\gamma_{Mo}} = 3507\ KN$$

Therefore $N_{Ed} = 141kn\ \leq N_{C,Rd}$

Bending and axial force interaction:

When axial force and bending moment act simultaneously on a cross section, the axial force can be ignored provided the following two conditions are satisfied.

ii) $V_{Ed} \leq 0.25N_{pl,Rd}$ *and* $ii)\ V_{Ed} \leq \frac{0.5h_w t_w f_y}{\gamma_{Mo}} = \frac{0.5X420.8X9.4X355}{1.0} = 702kn$

Therefore 132KN<877kn and 132Kn< 702kn OK

Therefore the effect of the axial force on the moment resistance may be neglected.

Bending moment resistance:

$$M_{pl,y,Rd} = \frac{1702X10^3X355}{1.0} = 604kn - m$$

Therefore 374 <604kn-m

Out-of plane of buckling:

The out-of plane buckling interaction is verified with the below expression

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

This expression should verified between torsional restraints.

If advantage is taken of intermediate restraints to the tension flange, the spacing of the intermediate restraints must be verified.

Mid-Span region: - The purlin spacing in this region is 1800mm

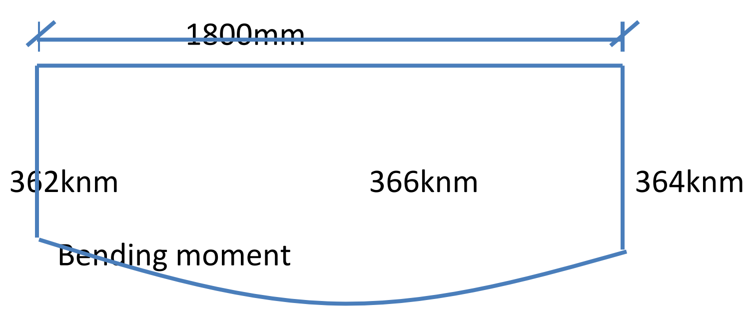


Fig.3.8. maximum Moment variation between purlins

Flexural buckling resistance about the minor axis bending, $N_{b,z,Rd}$:

$$\frac{h}{b} = \frac{450}{190} = 2.37, \qquad t_f = 14.6mm$$

Buckling about z-z axis,

Curve "b" for hot rolled "I" sections. The imperfection factor is $\alpha = 0.34$

$$\lambda_1 = 76.4$$

$$\bar{\lambda}_z = \frac{L_{cr}}{i_z}\frac{1}{\lambda_1} = \frac{1800}{41.2}\frac{1}{76.4} = 0.57$$

$$\phi_z = 0.5[1 + 0.34(0.57 - 0.2) + 0.57^2] = 0.726$$

$$\chi_z = \frac{1}{\phi_z + \sqrt{{\phi_z}^2 - {\bar{\lambda}_z}^2}} = \frac{1}{0.726 + \sqrt{0.726^2 - 0.57^2}} = 0.85$$

$$N_{b,z,Rd} = \frac{\chi_z A f_y}{\gamma_{M1}} = \frac{0.85X9880X355}{1.0}X10^{-3} = 2981kn$$

$N_{Ed} = 132kn < 2981kn$ OK

Lateral Torsional Buckling resistance, $M_{b,rd}$

In this zone , lateral torsional buckling is checked between restraints, which are the purlins. For equally spaced purlins, the critical length is at the point of maximum bending moment.

In order to determine the critical moment of the rafter, the C_1, factor takes account of the shape of the bending moment diagram.

In this case the bending moment diagram is nearly constant along the segment in consideration , so $\psi = 1$. Therefore $C_1 = 1.0$

$$M_{cr} = 1.0\frac{\pi^2 X210000X1676X10^4}{1800^2}X\sqrt{\frac{791X10^9}{1676X10^4}+\frac{1800^2X81000X66.9X10^4}{\pi^2X210000X1676X10^4}} = 2450kn - m$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{1702X10^3X355}{2450X10^6}} = 0.496$$

For hot rolled sections

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta\bar{\lambda}_{LT}{}^2\right]$$

$$\phi_{LT} = 0.5[1 + 0.49(0.496 - 0.4) + 0.75(0.496^2)] = 0.613$$

$$\chi_{LT} = \frac{1}{0.613 + \sqrt{0.613^2 - (0.75)0.496^2}} = 0.951 < 1.0$$

$$Therefore\ \chi_{LT} = 0.951$$

$$M_{b,Rd} = \frac{\chi_{LT} W_{pl,y} f_y}{\gamma_{M1}} = 573\ kn - m$$

$M_{Ed} = 374 < 573\ kn - m$ OK

Interaction of axial force and bending moment:

Out-of-plane Buckling due to the interaction of axial force and bending moment is verified by satisfying the following expression.

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

$\bar{\lambda}_z \geq 0.4$, the interaction factor, k_{zy} is calculated as ($\bar{\lambda}_z = 0.57$)

$$k_{zy} = max\left[\left(1 - \frac{0.1\bar{\lambda}_z}{C_{mLT} - 0.25}\frac{N_{Ed}}{N_{b,Rd,z}}\right); \left(1 - \frac{0.1}{C_{mLT} - 0.25}\frac{N_{Ed}}{N_{b,Rd,z}}\right)\right]$$

The bending moment is approximately linear and constant C_{mLT} is taken as 1.0.

Therefore $C_{mLT} = 1$

$$k_{zy} = max\left[\left(1 - \frac{0.1X0.57}{1.0 - 0.25}\frac{132}{2981}\right);\ \left(1 - \frac{0.1}{1.0 - 0.25}\frac{132}{2981}\right)\right]$$

$= max[0.997; 0.994]$=0.997

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{132}{2981} + 0.997\frac{374}{573} = 0.694 < 1.0$ OK

End of span region:

In this region the bottom flange is in compression and stability must be checked between torsional restraints.

The buckling length is taken from torsional restraint at sharp end of the haunch to the virtual restraint which is the point of contra flexure of the bending moment diagram, i.e, where the bending moment is equal to zero. If virtual restraint may not common practice, the buckling length should be taken to the next purlin

From the analysis , buckling length to the point of contra flexure is 2930mm

If the tension flange is restrained at discrete points between torsional restraints and the spacing between the restraints to tension flange is small enough, advantage may be taken of this situation.

In order to determine whether or not spacing between the restraints is small enough, Annex BB of EN1993-1-1 provides an expression to calculate the maximum spacing .If the actual spacing between restraints is smaller than this calculated value.

Verification of the spacing between intermediate restraints:

In this case, the restraint to the tension flange is provided by the purlins. Theses purlins are spaced at 1800mm

Therefore the limiting spacing is

$$L_m = \frac{38 i_z}{\sqrt{\frac{1}{57.4}\left(\frac{N_{Ed}}{A}\right) + \frac{1}{756 {C_1}^2} \frac{{W_{pl,y}}^2}{A I_t} \left(\frac{f_y}{235}\right)^2}} \quad \text{Eq-37}$$

C_1 −is a factor that account for the shape of the bending moment diagram. For linear bending moment diagram C_1 −depends on the ratio of the minimum and maximum bending moments in the segment being considered.

$$\psi = \frac{121}{298} = 0.406, for\ this\ the\ C_1 = 1.39$$

Therefore $L_m = 1669mm\ < purlin\ spacing\ \ 1800mm$

Therefore normal design procedure must be adopted and advantage may not be taken of the restraint to the tension flange.

Flexural Buckling resistance about the minor axis,$N_{b,z,Rd}$:

As previously

Buckling about Z-z axis: Curve “b” for hot rolled “I” sections, $\alpha_z = 0.34$

Therefore $\lambda_1 = 76.4$

$$\bar{\lambda}_z = \frac{L_{cr}}{i_z} \frac{1}{\lambda_1} = \frac{2930}{41.2} \frac{1}{76.4} = 0.931$$

$\bar{\lambda}_z = 0.931$ and

$\phi_z = 0.5[1 + 0.34(0.931 - 0.2) + 0.931^2] = 1.06$

$\chi_z = 0.638$

$$N_{b,z,Rd} = \frac{\chi_z A f_y}{\gamma_{M1}} = \frac{0.638X9880X355}{1.0} 10^{-3} = 2238kn$$

$N_{Ed} = 132kn < 2238kn$ OK

Lateral Torsional Buckling resistance$M_{b,Rd}$:

The lateral torsional buckling resistance of member is calculated as a reduction factor χ_{LT} multiplied by the section modulus and the yield strength of the section .The reduction factor is calculated as a function of the slenderness$\bar{\lambda}_{LT}$, which depends on the critical moment of the member. The expression for the critical moment M_{cr} is given below. The factor C_1 accounts for the shape of bending moment diagram of the member. For the case of a linear bending moment diagram, C_1 depends on the ratio of the bending moments at the ends of the member, given as ψ

For simplicity the bending moment diagram is considered as linear, which is slightly conservative.

$$\psi = \frac{0}{298} = 0 \quad for\ this\ \ value\ C_1 = 1.77$$

$$M_{cr} = C_1 \frac{\pi^2 E I_Z}{L^2} \sqrt{\frac{I_W}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z}}$$

$$M_{cr} = 1.77 \frac{\pi^2 X 210000 X 1676 X 10^4}{2930^2} X \sqrt{\frac{791 X 10^9}{1676 X 10^4} + \frac{2930^2 X 81000 X 66.9 X 10^4}{\pi^2 X 210000 X 1676 X 10^4}}$$

$$= 1763 X 10^6\ n - mm$$

The non-dimensional slenderness $\bar{\lambda}_{LT}$ is calculated as

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{2194 X 10^3 X 355}{1763 X 10^6}} = 0.585$$

$$\phi_{LT} = 0.5 \left[1 + \alpha_{LT} \left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta \bar{\lambda}_{LT}{}^2\right]$$

En 1993-1-1 recommends the following values:

$$\bar{\lambda}_{LT,o} = 0.4\ and\ \beta = 0.75$$

curve “c” for hot rolled I $sections$, Therefore $\alpha_{LT} = 0.49$

$$\phi_{LT} = 0.5[1 + 0.49(0.585 - 0.4) + 0.75(0.585)^2] = 0.674$$

$$\chi_{LT} = \frac{1}{\phi_{LT}+\sqrt{{\phi_{LT}}^2-\beta{\bar{\lambda}_{LT}}^2}} = \frac{1}{0.674+\sqrt{0.674^2-0.75X0.585^2}} = 0.894 \qquad \text{Eq-39}$$

Therefore $M_{b,Rd} = \frac{\chi_{LT}W_{pl,y}f_y}{\gamma_{M1}} = \frac{0.894\ X1702X10^3X355}{1.0}X10^{-6} = 540\ kn - m$

$$M_{b,Rd} = 298 < 540\ kn - m \qquad OK$$

Interaction of axial force and bending moment:

Out-of-plane Buckling due to the interaction of axial force and bending moment is verified by satisfying the following expression.

$$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

$\bar{\lambda}_z \geq 0.4$, the interaction factor ,k_{zy} is calculated as

$$k_{zy} = max\left[\left(1 - \frac{0.1\bar{\lambda}_z}{C_{mLT} - 0.25}\frac{N_{Ed}}{N_{b,Rd,z}}\right);\left(1 - \frac{0.1}{C_{mLT} - 0.25}\frac{N_{Ed}}{N_{b,Rd,z}}\right)\right]$$

$$\psi = \frac{0}{298} = 0$$

$$C_{mLT} = 0.6 + 0.4\psi = 0.6$$

Therefore $C_{mLT} = 0.6$

$$k_{zy} = max\left[\left(1 - \frac{0.1X0.931}{0.6 - 0.25}\frac{132}{2238}\right);\ \left(1 - \frac{0.1}{0.6 - 0.25}\frac{132}{2238}\right)\right]$$

$= max[0.984; 0.983]$=0.983

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{132}{2238} + 0.983\frac{298}{540} = 0.601 < 1.0$ OK

In – plane Buckling :

The in plane buckling interaction is verified with the following expression

$$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

Maximum bending moment and axial force in the rafter, excluding the haunch

$M_{Ed} = 374kn - m$

$N_{Ed} = 132kn$

Flexural buckling resistance about the major axis, $N_{b,y,Rd}$:

$$\frac{h}{b} = \frac{450}{190} = 2.37$$

$$t_f = 14.6mm$$

Buckling is about y-y axis.

Curve "a" for hot rolled "I" Sections

$$\alpha_y = 0.21$$

The buckling length is the system length, which is the distance between joints(i.e, the length of the rafter including the haunch), L=20300mm

$$\lambda_1 = 76.4$$

$$\bar{\lambda}_y = \frac{L_{cr}}{i_y}\frac{1}{\lambda_1} = \frac{20300}{185}\frac{1}{76.4} = 1.436$$

$$\phi_y = 0.5[1 + 0.21(1.436 - 0.2) + 1.436^2] = 1.66$$

$$\chi_y = \frac{1}{1.66 + \sqrt{1.66^2 - 1.436^2}} = 0.201$$

$$N_{b,Z,Rd} = \frac{0.401X9880X355}{1.0} X10^{-3} = 1407KN$$

$$N_{Ed} = 132kn < 1407\ kn \qquad OK$$

Lateral Torsional Buckling Resistance,$M_{b,Rd}$:

$M_{b,Rd}$ − is the least buckling moment resistance of those calculated previously

$$M_{b,Rd} = \min(581; 540)$$

Therefore $M_{b,Rd} = 540\ kn - m$

Interaction of axial force and bending moment:

In-Plane buckling due to the interaction of axial force and bending moment is verified by satisfying the following expression.

$$\frac{N_{Ed}}{N_{b,y,Rd}} + k_{yy} \frac{M_{y,Ed}}{M_{b,Rd}} \leq 1.0$$

the interaction factor ,k_{yy} is calculated as follows.

$$k_{yy} = min\left[C_{my}\left(1 + (\bar{\lambda}_y - 0.2)\frac{N_{Ed}}{N_{b,y,Rd}}\right); C_{my}\left(1 + 0.8\frac{N_{Ed}}{N_{b,y,Rd}}\right)\right]$$

C_{my} −is depends on the values of $\alpha_h\ and\ \psi$

$$\psi = -\frac{298}{362} = -0.82$$

$$\alpha_h = \frac{M_h}{M_s} = \frac{362}{374} = 0.97$$

Therefore C_{my} is calculated as

$$C_{my} = 0.95 + 0.05\alpha_h = 0.95 + 0.05X0.97 = 0.95$$

Therefore $C_{mLT} = 0.95$

$$k_{zy} = min\left[0.95\left(1 + (1.436 - 0.2)\frac{132}{1407}\right); 0.95\left(1 + 0.8\frac{132}{1407}\right)\right]$$

$= min[1.06; 1.021]$=1.021

$\frac{N_{Ed}}{N_{b,z,Rd}} + k_{zy}\frac{M_{y,Ed}}{M_{b,Rd}} = \frac{132}{1407} + 1.021\frac{374}{540} = 0.808 < 1.0$ OK

The member satisfies the in plane buckling. Check.

Validity of the rafter section:

It has been demonstrated that the cross sectional resistance of the section is greater the applied forces.

The Out-of-Plane and In-Plane buckling checks have been verified, for the appropriate choice of restraints along the rafter

Therefore IPE 600 section S355 steel is appropriate for use as rafter in this portal frame.

Haunched length:

The haunch is fabricated from a cutting of an IPE500 section. Checks must be carried out at end and quarter points as executed above.

3.3 Plastic Analysis:

In more complex frames it is convenient to use the instantaneous Centre of rotation method when developing collapse mechanism. In general the number of hinges required to convert the portal to a mechanism is one more than the statical indeterminacy. With unsymmetrical loads such as dead load Plus wind load two hinges only form to cause collapse.

For the location of hinges to be correct, the plastic moment at the hinges must not be exceeded at any point in the structure, that is why two plastic hinges form at each side of the ridge and not one only at the ridge at collapse. The critical mechanism is the one which gives the lowest value for the collapse load. The collapse mechanism which occurs depends on the form of loading. Plastic analysis for the pinned base portal is carried out in the following two stages.

i) The frame is released to a statically determinate state by inserting rollers at one support
ii) The free bending moment diagram is drawn
iii) The reactant bending moment diagram due to the redundant horizontal reaction is drawn.
iv) The free and reactant bending moment diagrams are combined to give the plastic Bending moment diagram with sufficient hinges to cause the frame or part of it(Example: Rafter) to collapse

The exact location of the hinge near the ridge must be found by successive trials or mathematically if the loading is taken to be uniformly distributed.

If the load is taken to be applied at the purlin points, the hinge will occur at a purlin location. The purlin may be checked in turn to see which location gives the maximum value of the plastic moment.

Summary of design procedure:

1) Select steel grade and trial sections
2) Check in plane stability of frame $\left(\lambda_p \geq \lambda_r\right) using$:

Sway check method

Amplified moments method or

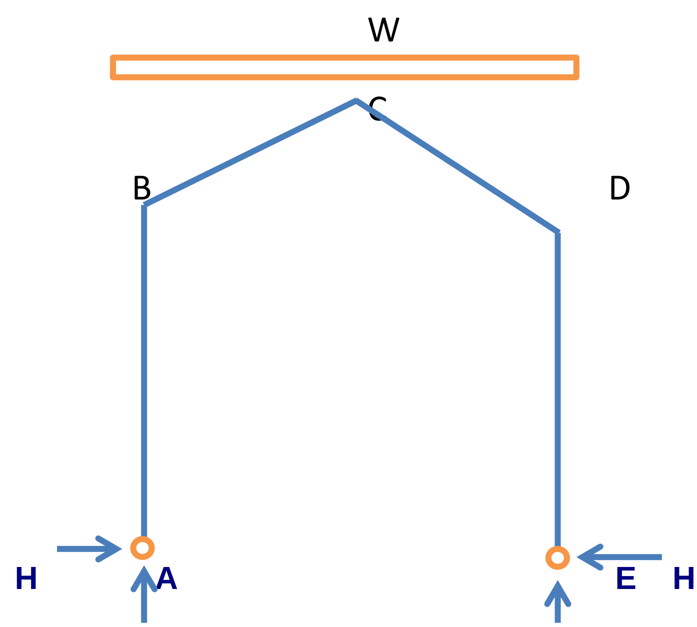
Second order analysis

3) Check out of plane stability of frame
4) Check in –plane stability of members
5) Check out of plane stability of members

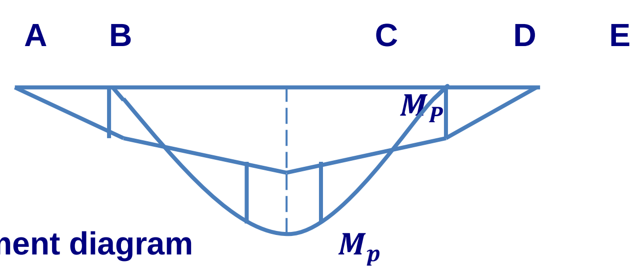
    Determine limiting segment length for

    a) Segment adjacent to plastic hinge (L_m)
    b) Member or segment with one flange restrained $(L_s)\ using$
        - Simple method or
        - Annex G approach

6) Check deflections
7) Design connections and bases to transmit forces and moments.

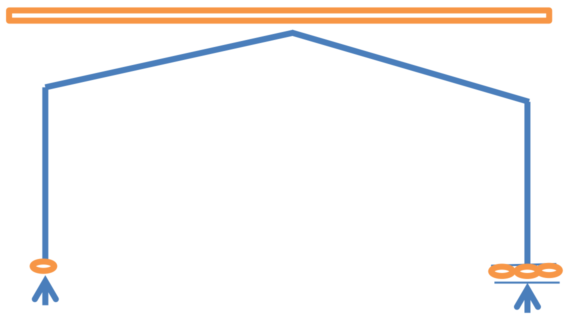


Frame And Loads

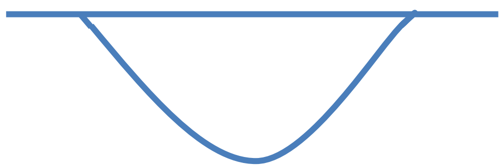


Plastic moment diagram

Fig.3.9 Combined bending moment diagram

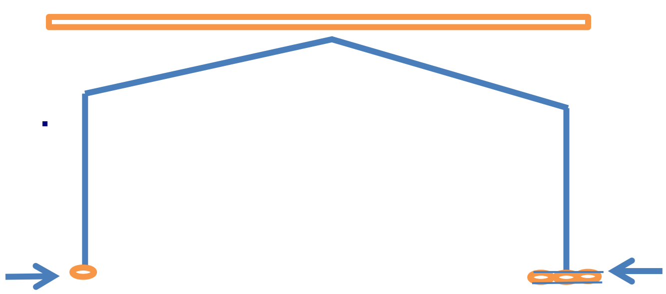


RollersReleased Frame.



Frame moment diagram

Fig.3.10 Free bending moment diagram



Redundant

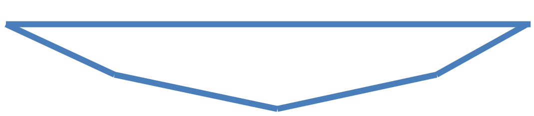


Fig.3.11. Reactant Bending Moment diagram.

Section design :

At hinge locations design is made for axial load and plastic moment. The following two design procedures can be used. Simplified method and Exact method In the exact method axial load reduces the plastic moment of resistance of a section. The bending moment is resisted by two equal areas extending inwards from the edges. The central area resists axial load and this area may be confined to the web or extended into the flange under heavy load. The stress diagrams are shown below.

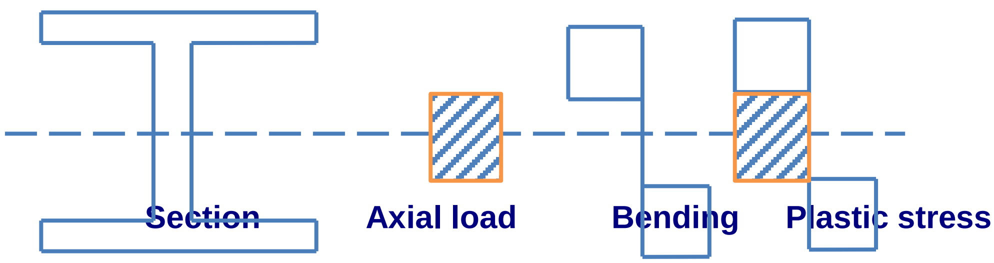


Fig.3.12. Load resistance contributing areas.

The formulae for calculating the reduced plastic moduli of sections subjected to axial load and moment are given in steel work design to BS 5950 , volum-1 and the calculation procedure is as fllows.

$n = {}^{F}/_{A_g P_y}$ Where $F -$ Applied axial load, $A_g -$ gross area. Eq-40

For lower values of "n" the neutral axis lies in the web and the reduced plastic modulus

$S_R = k_1 - k_2 n^2$ Eq-41

Reduced moment capacity $M_R = S_R P_y > M$ Eq-42

The analysis for finding the plastic moment in the column and rafter is done in two stages i) Dead load+ Imposed load and ii) Dead load + Imposed load + Wind load.

Case-i) Dead load+ imposed load

The plastic moment moments in the column at the bottom of the haunch and in the rafter at "x" from the eaves are given by the following expressions.

D.L+I.L= 10kn/m

$M_p = 7.275H$

The rafter moment capacity lies between 60 to 75%. Therefore choose the rafter moment capacity is 75% of the column moment capacity.

$$0.75M_p = 200x - 10\frac{x^2}{2} - H(8 + 0.176x)$$

Therefore $H = \frac{200-x^2}{(13.45+0.176x)}$

Do $\frac{dH}{dx} = 0$ and fine the "x" value.

Therefore $x = 17.9\ m$

From the known "x" value $H = 119.1KN\ and\ M_p = 866\ kn - m$ in the column at the bottom of the haunch and the moment in the rafter is 0.7 $= 89.25kn - m$

Case-ii) Dead Load+ Imposed Load +wind load

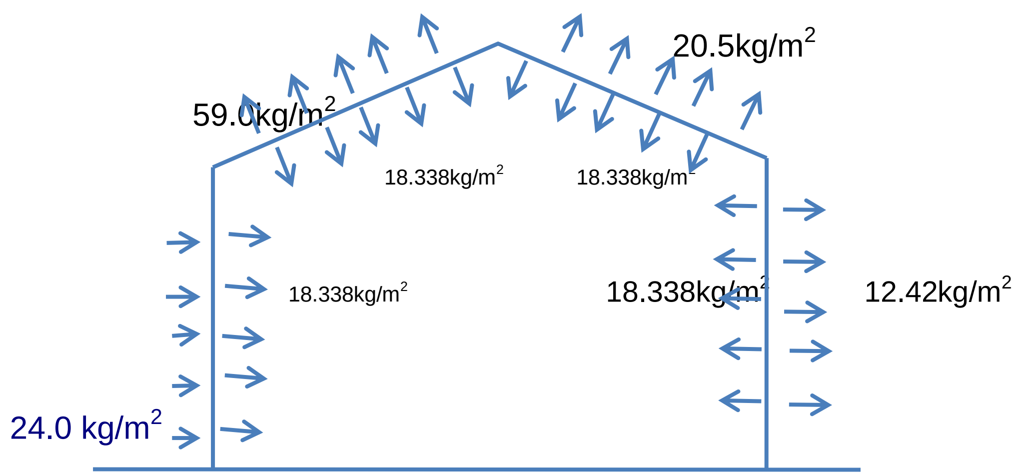


Fig.3.13. Internal and external pressures on various surfaces

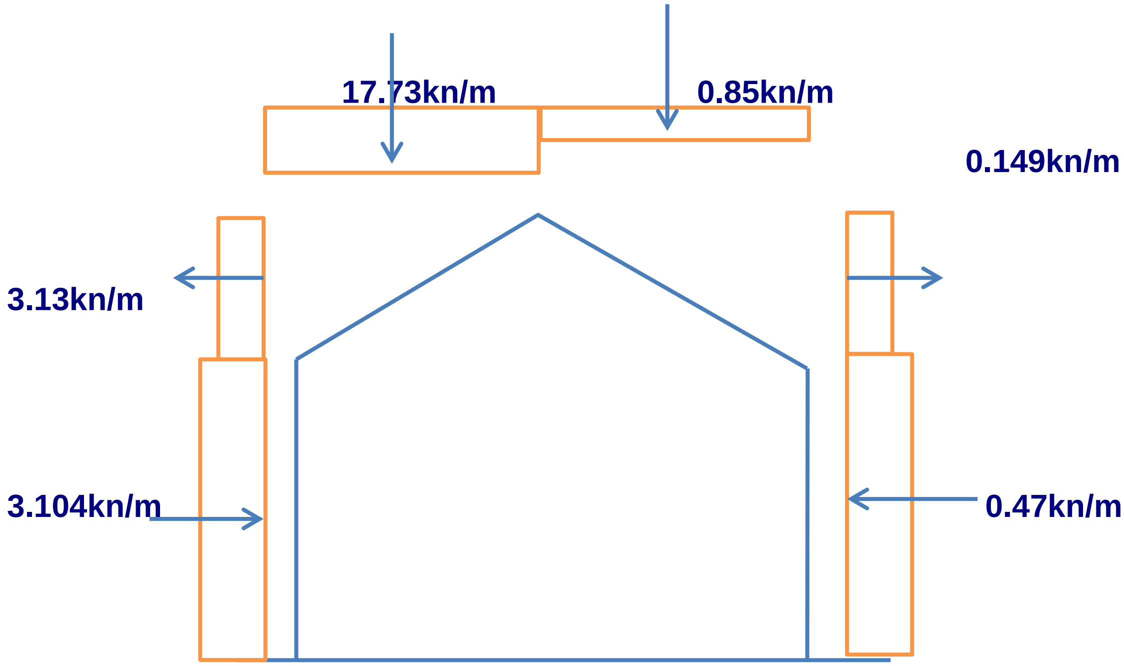


Fig.3.14 Wind pressure distribution.

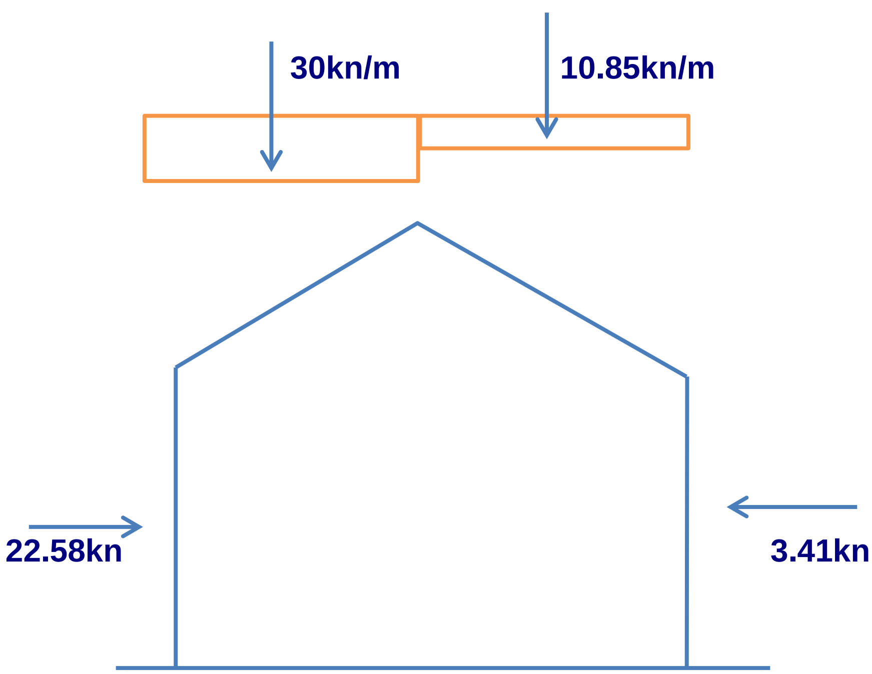


Fig.3.15 D.L+L.L+IL combination

$\boldsymbol{M_x = 496(x) - 69.66 - 30\frac{x^2}{2}}$

At eves level below the haunch where the hinge is formed is showed in below figure $M_P = 7.275\,H + 12.437$

$$0.75M_P = 496(x) - 69.66 - 15x^2 - H(8 + 0176x)$$

$$H = \frac{496(x) - 78.99 - 15x^2}{13.456 + 0.176x}$$

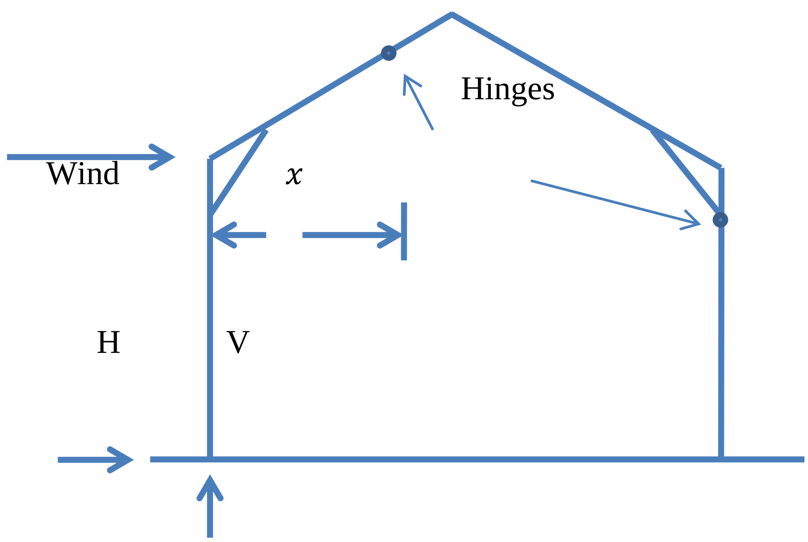


Fig.3.16 Plastic analysis case with wind load

Do $\frac{dH}{dx} = 0$ find "x" value

Therefore $x = 18.9m$

$H = 234Kn$ and $M_p = 1714\;kn - m$ for column and for the rafter the moment is $0.75M_p = 1285.5\;kn - m$

Therefore the result of the D.L+L.L+W.L combination have the more forces than D.L+ L.L combination. Us the values of the former case for doing the design of the frame section.

Column section:

The design action at the column hinge:

$M_p = 1714\ kn - m$

F = 822.72kn

$$section\ modulas\ required\ for\ this\ load\ is\ = \frac{M_P}{f_y} = \frac{1714X10^6}{355} = 4.83X10^6\ mm^3$$

Choose the IPE 750X147

$$W_{ply} = 5110X10^3 mm^3, h = 753mm, b - 265mm, t_w = 13.2mm, t_f = 17mm$$

$$r = 17mm, A = 188X10^2\ mm^2, I_y = 166100X10^4 mm^4, W_{ely} = 4411X10^3\ mm^3\ i_y = 29.76$$

$$I_z = 5289X10^4 mm^4, W_{elz} = 399.2X10^3 mm^{3,} W_{plz} = 630.8X10^3 mm^3, i_z = 5.31mm$$

$$n = \frac{822.72X10^3}{188X10^2X355} = 0.012 < 0.444$$

Reduced plastic modulus : $S_R = 5110X10^3 - 1260X10^3 n^2 = 510.98X10^3$

The section is satisfactory

Rafter section :

$M_p = 1285.5\ kn - m$

$F = 365kn$

$$Section\ modulud\ required\ is = S = \frac{1285.5X10^6}{355} = 3.62X10^6 mm^3$$

Choose IPEo600:

Properties of the section:

$$W_{ply} = 4471X10^3 mm^3, h = 610mm, b = 224mm, t_w = 15mm, t_f = 24mm$$

$$r = 24mm, A = 197X10^2\ mm^2, I_y = 118300X10^4mm^4, W_{ely} = 3879X10^3\ mm^3\ i_y = 24.52$$

$$I_z = 4521X10^4mm^4, W_{elz} = 403.6X10^3mm^{3,}W_{plz} = 640.1X10^3mm^3, i_z = 4.79mm$$

$$n = \frac{365X10^3}{197X10^2X355} = 0.05 < 0.437$$

$$S_R = 4471X10^3 - 1260X10^3n^2 = 4467.85X10^3mm^3$$

The section is satisfactory

Check that the rafter section at the end of the haunch remains elastic under factored loads. The actions are

Maximum stress $= \frac{422X10^3}{197X10^2} + \frac{980X10^6}{4471X10^3} = 240.42 < 355$n/mm2

Therefore the section is elastic.

Column restrains and stability:

A tie is provided to restraint the hinge section at the eaves. The distance to the adjacent restraint using the conservative method is.

$$L_m \leq \frac{38 * 24.52}{\frac{365X10}{422X130} + \left(\frac{42}{36}\right)^2} = 653\ mm$$

$$f_c - compressive\ stress\ due\ to\ axial\ load.$$

$r_y - radius\ of\ gyration\ about\ YY\ axis$ Using the minimum value if the section varies.

$x -$ Torsional index using the maximum value if the section varies.

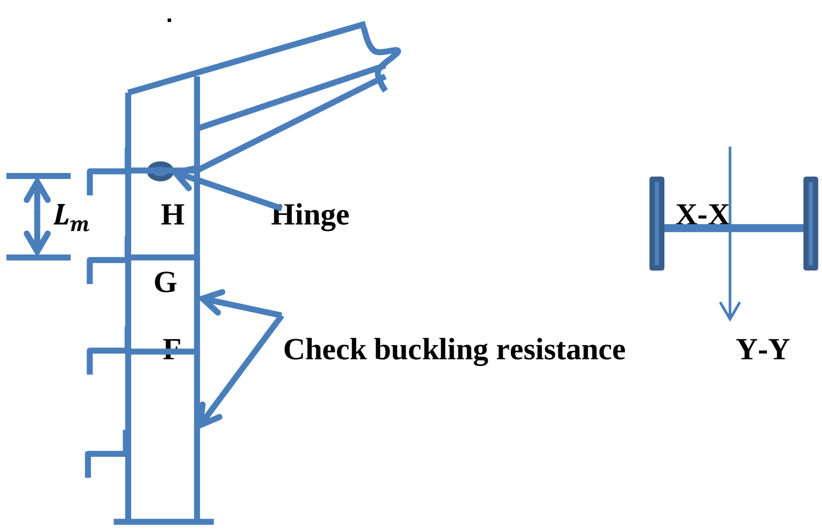


Fig. 3.17. Hinge formation

When this method is used no further checks are required. The locations of the restraints at hinge H, and at G, L_m below H are shown in figure. It may be necessary to introduce a further restraint at F below G, in which case column lengths GF and FA would be checked for buckling resistance for axial load and moment. The effective lengths for XX- axis may be estimated for the portal, the steel work design guide to BS5950 takes the effective length for the XX-axis as HA as shown in the above figure, i.e, the distance between the plastic hinge and base. The effective lengths for YY axis are GF and FA. Note that compliance with the sway stability check ensures that the portal can safely resist in-plane buckling and additional moments due to frame deflections.

Over all buckling check value is less than one , therefore it is satisfactory. Note that the in-plane stability or sway stability according to the BS 5950.

Rafter restraints and stability:

The rafter section at the eaves. The center flange is neglected and the section properties are calculated to give. The rafter stability is checked taking account of restraint to the tension flange

Check the interaction expression, at the large end of the haunch, at the eaves consider the welded section and at the small end of the haunch

The interaction expression $F/P_c + M_x/M_b \leq 1$, therefore this expression is satisfied at large end and also at the smaller end of the haunch . The haunch is satisfactory.

If the rafter is checked using the in plane effective length derived in the elastic design earlier the slenderness $L_{EX}/r_X > 180$, and its depth would need to be increased if this more conservative design procedure is used: However, the rafter is primarily a flexural member carrying small axial load and the restriction $L/r < 180$, intended for struts should not, in fact apply

Chapter 4

Results

4.1 Results:

It is proved that the same member under the elastic condition carried less load compared to the plastic condition. The increase in the moment in case of rafter is 343.5% increment and shear is about 276.5% increment. For the column member the moment increment is 263.7% and for the column the shear increment is 703.16%

Second order effects are not dealt separately, it will be taken care by the method itself. That is $\alpha_{cr}\ facotr(in\ plane\ flexibility\ of\ the\ frame)$ allowable limit is more in plastic analysis($\geq 15\%$) compared to the elastic analysis.

It also proved that the analysis and design of the gable frame with plastic method is efficient and economy in the sense that maximum capacity of the section has utilized and this method will take less time compared to the elastic analysis.

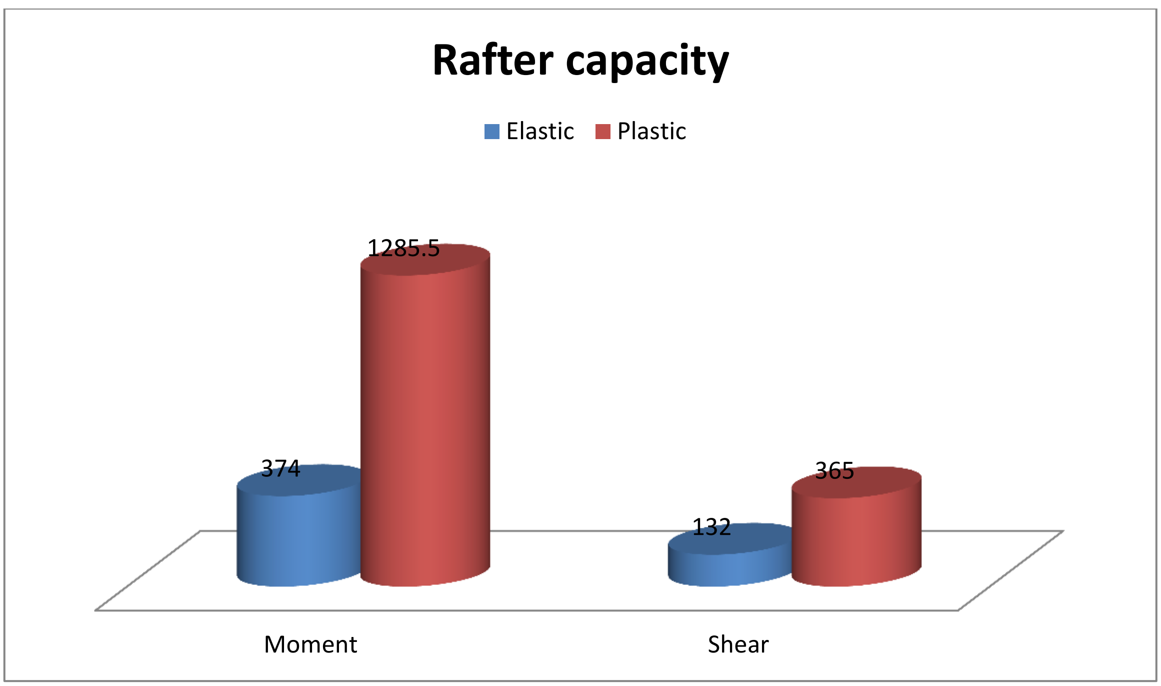


Fig 4.1.Increment in moment and shear from elastic to plastic for rafter

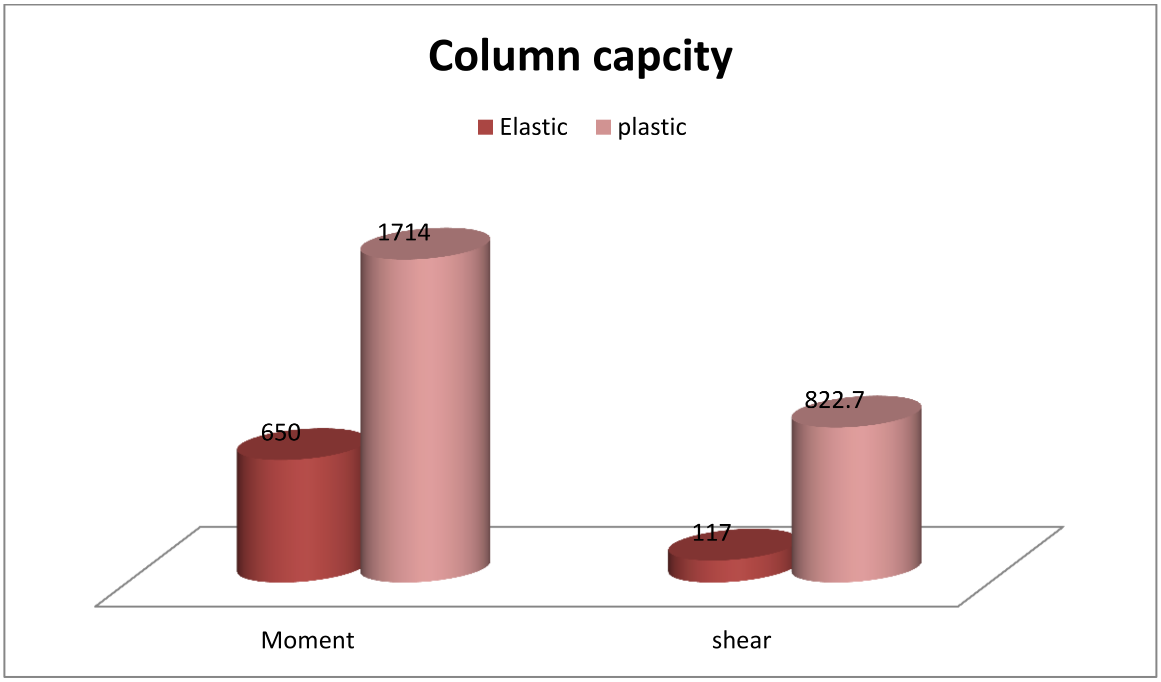


Fig 4.2. Increment in moment and shear from elastic to plastic for the column

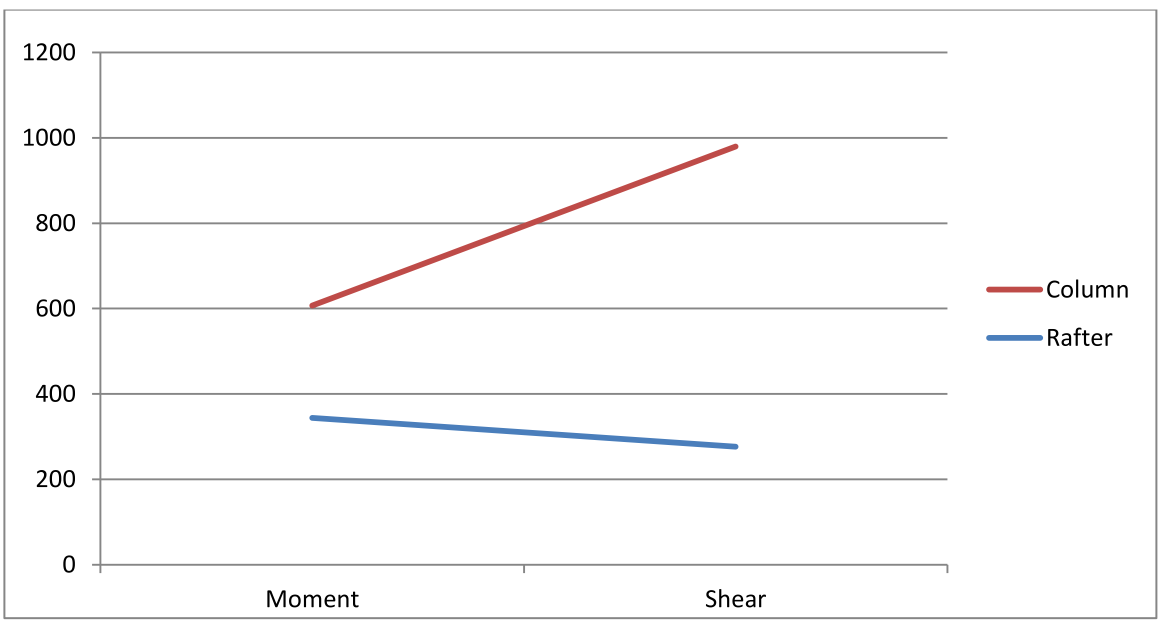


Fig.4.3 Percentage increment in moment and shear

<u>Cost Analysis:</u>

The size of the section suitable for carrying the loads of elastic analysis in plastic analysis is IP400 for the rafter and IPE 450 for the column section. The cost of steel per ton in Euro is considered here

The cost analysis is for a gable frame because the difference in element sizes comes in the frame according to the plastic and elastic analysis

S.No	Size of the member	Weight of the member Kg/m	Length of the members (m)	Total weight of the member Kg	Total weight in tons	Cost of the member per ton	Total cost Euro	Cost of the singe Gable frame
Plastic Analysis								
1.	IPE-400	66.7 Kg/m	20.37	1353.8	1.353	565 Euro/ton	764.00	1131.38
2	IPE-450	77.6 Kg/m	8m	620.8	0.628	585 Euro/ton	367.38	
Elastic Analysis								
3	IPE-0600	154Kg/m	20.37	3136.98	3.137	595 Euro/ton	1866.515	2695.595
4	IPE-750X147	147Kg/m	8m	1176	1.176	705 Euro/ton	829.08	

Table 4.1 Cost Analysis of a Gabled frame

Type of section	Cost per ton in Euro
IPE 330-400	565 Euro/ton
IPE 450-500	585Euro/ton
IPE 550-600	595 euro/ton
IPE 750	705 euro/ton
Source :Arcelor Mittal Europe – Long products sections and Merchant bars, Price list for Beams, channels and Merchant bars at the base of 2016 year	
Table 4.2 Price list from Arcelor Mittal	

4.2 Conclusion and Discussion:

i) As the plastic portion moves further into the beam toward the neutral axis , the beam will continue to resist the bending moment although with an increasing rate of deflection

ii) The factor of safety has more real meaning than at present because by plastic analysis which facilitates to determine the real maximum strength of the structure. It is not unusual for the factor of safety to vary from 1.65 up to 3 or more for structures designed according to conventional elastic methods.

iii) Using plastic analysis always gives less sections and less cost than elastic analysis. The economy of the plastic analysis also depends on the bracing system, because plastic redistribution imposes additional requirements on the restraint to members.

iv) For plastic design , all sections containing plastic hinges must be class-1

v) It is recognized that some redistribution of moments is possible, even with the use of elastic design.

vi) Rafters should be IPE or similar sections with class-1 or class-2 proportions under combined moment and axial load .Sections containing plastic hinges must be class-1

vii) Stays are required to stabilize inner flange of the columns and rafters where they are in compression and potentially unstable.

viii) At or near the plastic hinge positions to provide torsional restraint

4.3 Future scope of Work:

i) The plastic analysis can also be applied in the analysis of residential and commercial buildings

ii) The analysis and design of the Gable frame can be written in computer programming.

iii) It also makes the comparison of analysis based on the Rigid plastic analysis and elasto-plastic analysis to find effectiveness and efficient

iv) By using Bespoke software the $M - \emptyset$ curves can be developed in order to know the plastic plateau of the element chosen.

References:

i) EBCS-3-1995, Ethiopian Building code standard for steel structures

ii) EBCS-3-2013, Ethiopian Building code standard for steel structures

iii) EN-1993-1-1: Eurocode-3 Design steel structures, General rules and rules for building

iv) T.J.Macginley & T.C.Ang, structural steel work, Design to limit state theory.2001

v) Baker, J.F., Horne, M.R. and Heyman, J., The Steel Skeleton, Volume II, Plastic Behaviour and Design, Cambridge University Press, 1956.

vi) Baker, J.F. and Heyman, J., Plastic Design of Frames, Vol. 1: Fundamentals, Cambridge University Press, London, 1969.

vii) Bruneau, M., Uang, C.M. and Whittaker, A., Ductile Design of Steel Structures, McGraw-Hill, New York, 1998.

viii) Davies, J.M. and Brown, B.A., Plastic Design to BS5950, Blackwell Science, Oxford, 1996. • Heyman, J., Plastic Design of Portal Frames, Cambridge University Press, London, 1957.

ix) Heyman, J., Plastic Design of Frames, Vol. 2: Applications, Cambridge University Press, London, 1971.

x) McKenzie, W.M.C., Examples in Structural Analysis, Taylor and Francis, Abington, 2006.

xi) Neal, B.G., Structural Theorems and their Applications, Pergamon Press, 1964.

xii) Thompson, F., and Haywood, G.G., Structural Analysis Using Virtual Work, Chapman and Hall, 1986.

xiii) Rees, D.W.A., Mechanics of Solids and Structures, Imperial College Press, London, 2000.

xiv) Wong, M.B., Plastic Analysis and Design of Steel Structures, Butterworth-Heinemann, London, 2009

xv) HORNE, M.R AND AJMANI.J.L, failure of columns laterally supported on one flange: Discussion The structural Engineer, Vol.51.No.7, july,1973

xvi) LIM,J,KING,C.M,RATHBONE ,A,DAVIS,J.M and EDMONDSON,V Euro code -3:The in plane stability of portal frames, The structural Engineer, vol.83.No.21,1st November 2005

xvii) FISHER.J.M. Industrial buildings , Chapter 6.1 in construction steel design :an international guide , Elsevier Applied science, London, 1992.

xviii) Steel building in Europe, single story steel building part-2:concept design.

xix) HORNE M.R, Plastic theory of structures, pergamon press, oxford ,1979

xx) SCROSBY LOCKWOOD , LONDON, Steel Designers manual, London,1972

xxi) GHALI,A. and NAVILLE,A.M., structural Analysis, chapman and Hall, London.1978.

xxii) PASK, J.W., Manual on connections for beam and column construction, BCSA, London, 1982

xxiii) HORNE, M.R., and MORRIS, L.J., Plastic design of low rise frames, Collins, London,1981.

xxiv) TRAHAIR, N.S, The behavior and design of steel structures. Chapman and Hall, London, 1977.

Appendix

Rigid –Plastic Analysis

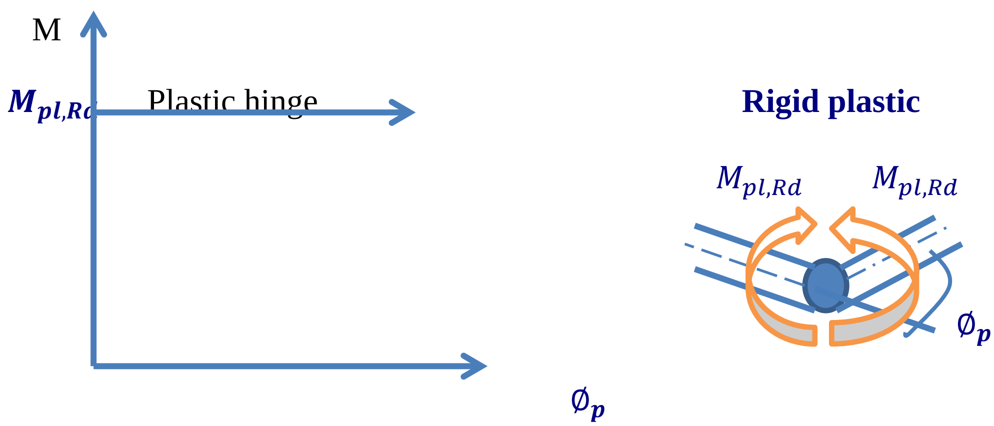


Fig A.1 Moment Rotation characteristics of the member.

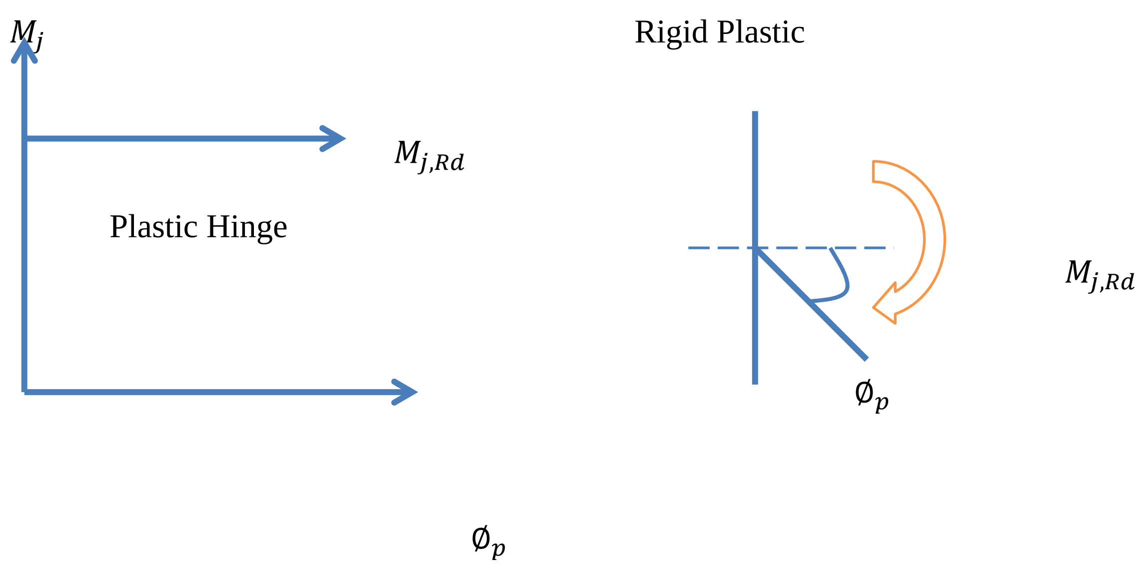


Fig A.2Moment Rotation characteristics of the joint

Elastic –perfectly plastic:

Elastic-Perfectly plastic

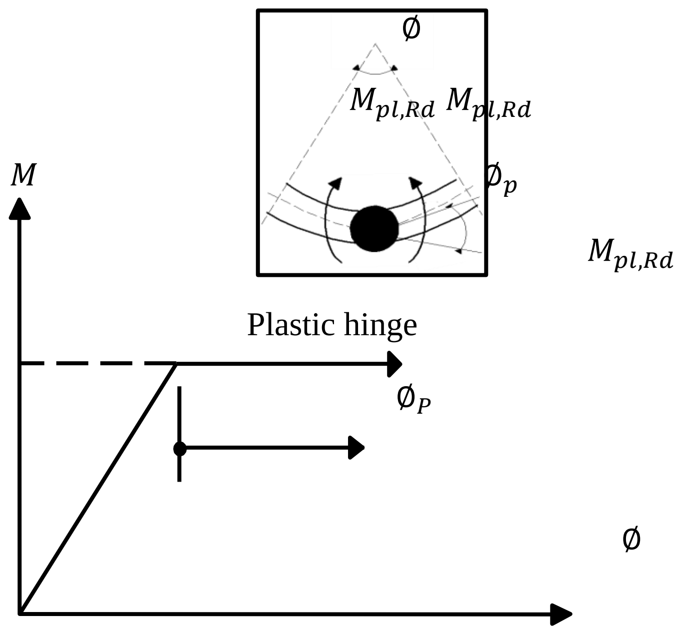


Fig A.3 Moment rotation characteristics of the cross section

Elastic –Perfectly plastic

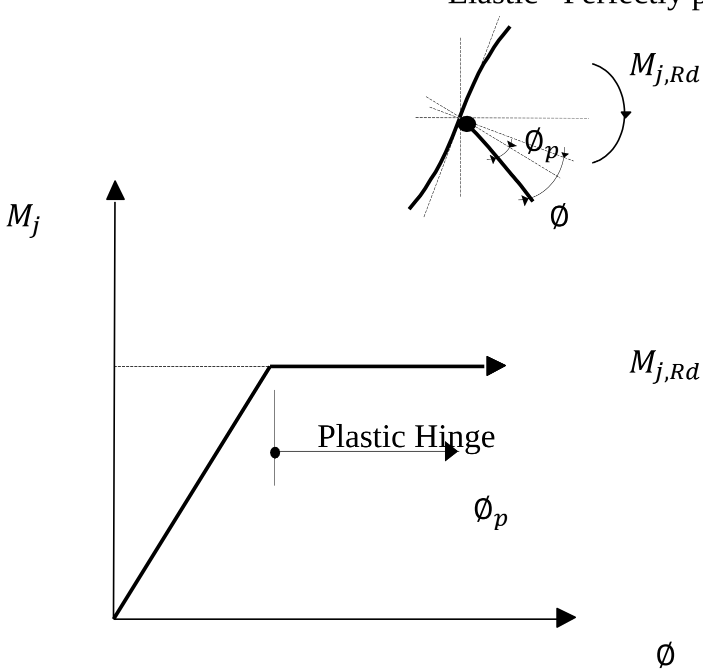


Fig A.4 Moment rotation characteristic of the joint

Elastic-plastic analysis:

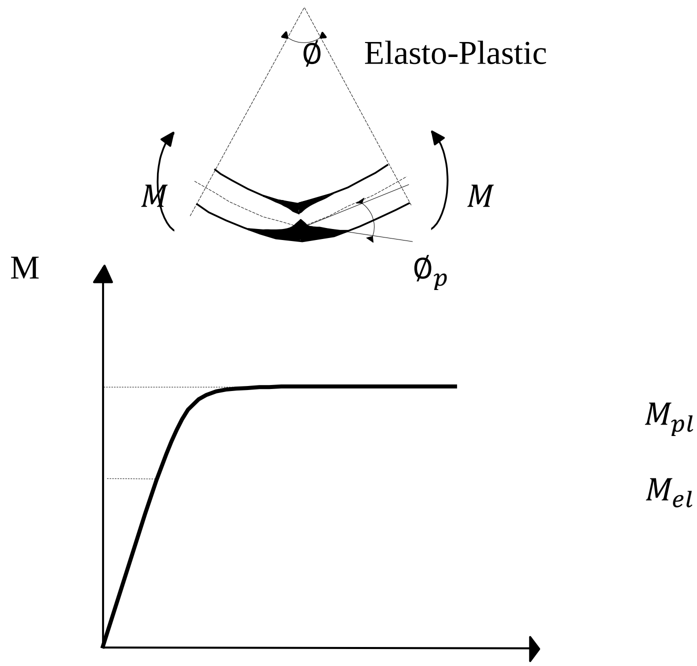


Fig A.5 Moment rotation characteristics of the member

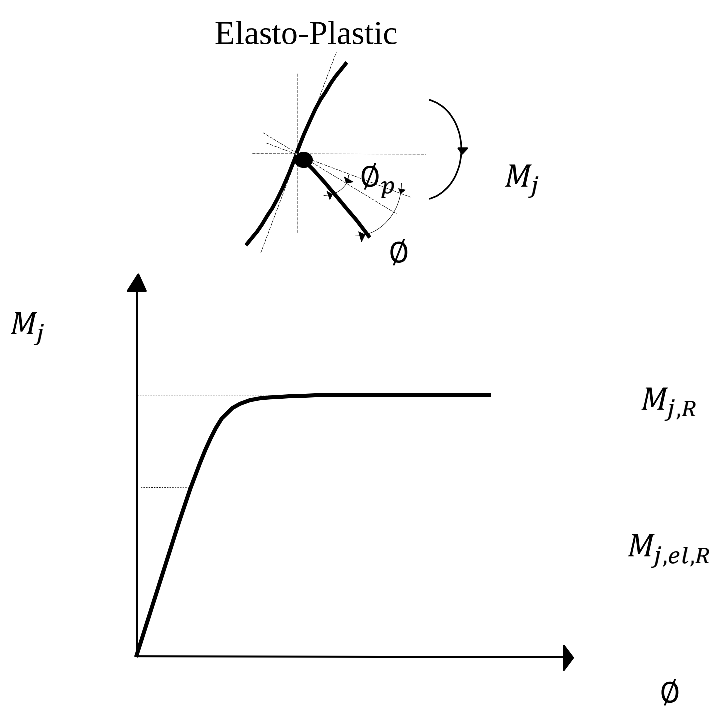


Fig A.6 Moment rotation characteristics of the joint

Elastic Analysis

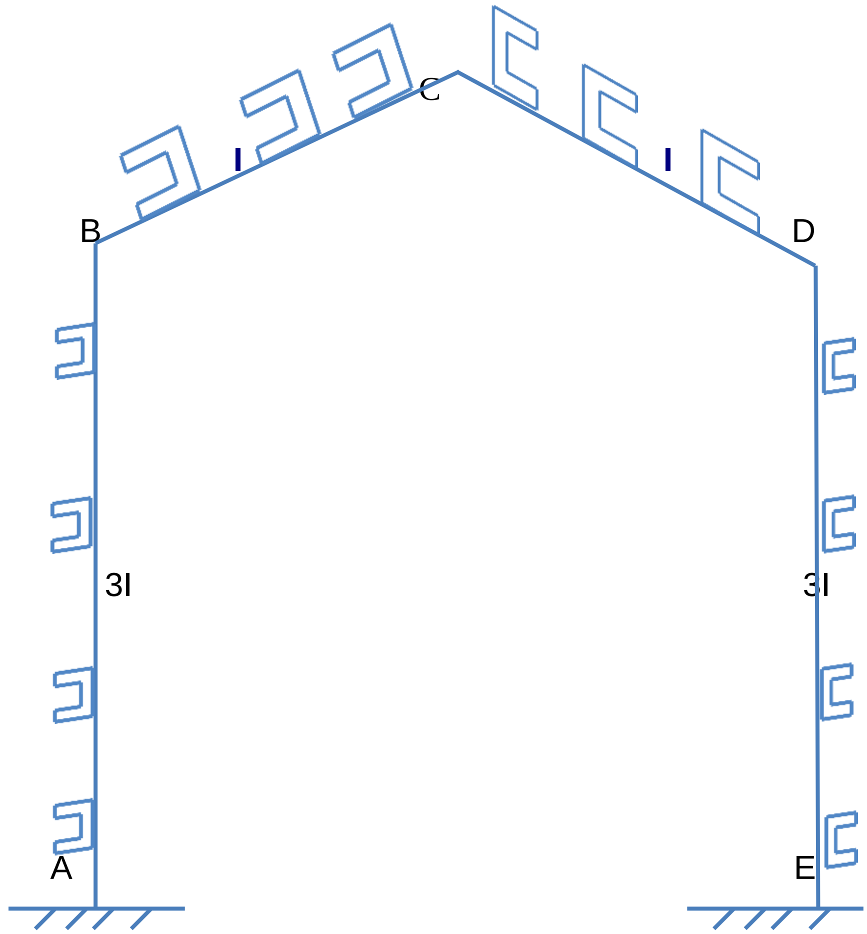


Fig A.7 Components of the Gable Frame

Fixed end moments

Member	FEM(Kn-m)
M_{FAB}	0
M_{FBA}	0
M_{FBC}	-122.5
M_{FCB}	122.5
M_{FCD}	-122.5
M_{FDC}	122.5
M_{FDE}	0
M_{FFD}	0

Table A.1 Fixed end moments

Distribution Factor:

Joint	Member	R.S	T.S	D.F
B	BA	3I/7	0.56I	0.765
	BC	I/ 7.616		0.234
C	CB	I/7.616	0.26I	0.5
	CD	I/7.616		0.5
	DC	I/7.616	0.56I	0.234
	DE	3I/7		0.765

Table A.2 Distribution Factors

Moment distribution:

Final Moment (Kn-m)	Carry over	Balancing	F.E.M	D.F	Member	Joint
46.858	46.858	-	-	-	AB	A
93.715	-	93.715	-	0.765	BA	B
-93.835	-	28.665	-122.5	0.234	BC	
136.833	14.333	-	122.5	0.5	CB	C
-136.833	-14.333	-	-122.5	0.5	CD	
93.835	-	-28.665	122.5	0.234	DC	D
-93.713	-	-93.715	-	.765	DE	
-46.858	-46.858	-	-	-	ED	E

Table A.3 Moment distribution